UPPSALA UNIVERSITET
Matematiska institutionen
Christer Kiselman

ÖVNINGAR, Blad 3
Transformer för beräkningar
2001 04 16 01trp3

3.1. Let a and b be two vectors in $l^2(\mathbf{Z}_3)$, a given by its Fourier transform $\hat{a} = (1, 2, 0)$, and $b = (1, \omega, \omega^2)$, where $\omega = e^{-2\pi i/3} = -\frac{1}{2} - \frac{i}{2}\sqrt{3}$.

(a) Calculate $\hat{b}$.

(b) Calculate $a * b$. Tenta 2000 05 22:1

3.2. Study the convolution equation $b * z = w$, where b is a given vector in $l^1(\mathbf{Z})$ (the black box), z is the input signal and w is the output signal. The problem is to decide which output signals w are possible with certain black boxes b.

(a) Let $b = \delta_0 + \delta_1 + \delta_2$, $w = \delta_0 + \delta_1$. Prove that there is no solution z to the equation $b * z = w$ such that $z(j) = 0$ for all large $|j|$.

(b) Now let $b = \delta_0 + \delta_1$, $w = \delta_0 + 2\delta_1 + \delta_2$. Prove that there exists an input signal z such that $b * z = w$ and such that $z(j) = 0$ for all large values of $|j|$.

(c) Finally let $b = \delta_0 + \delta_1$, $w = \delta_0$. Find all solutions $z \in l^\infty(\mathbf{Z})$ to $b * z = w$.

Tenta 2000 05 22:2

3.3. Suppose we know that $b \in l^2(\mathbf{Z}_6)$ satisfies $\hat{b}(0) = 0$, $\hat{b}(k) \neq 0$, $k = 1, ..., 5$.

(a) What is the dimension of the space of input signals producing the zero output signal, in other words, what is the dimension of the space

$$\{z \in l^2(\mathbf{Z}_6);\ b * z = 0\}?$$

(b) Determine the dimension of all possible output signals, i.e., the dimension of the space

$$\{b * z;\ z \in l^2(\mathbf{Z}_6)\}.$$

Tenta 2000 05 22:3

3.4. Let $u = (2, 0, 1)$, $v = (0, 4, 3)$, $z = (2, 0, 0, 4, 1, 3)$. Compute $\hat{u}$, $\hat{v}$ and then $\hat{z}$ using the fast Fourier transform. Tenta 2000 05 22:4

3.5. Prove that the convolution equation $a * z = \delta$ cannot be solved with $z \in l^2(\mathbf{Z})$ if $a(0) = a(1) = 1$ and $a(k) = 0$ for $k \in \mathbf{Z}$, $k \neq 0, 1$. Here δ is the Dirac δ, the vector with 1 at position zero and 0 at all other slots. Are there solutions in some other space $l^p(\mathbf{Z})$?