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Image enhancement and thresholding by optimization of fuzzy compactness

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Abstract: Algorithms based on minimization of compactness and of fuzziness are developed whereby it is possible to obtain both fuzzy and nonfuzzy (thresholded) versions of an ill-defined image. The incorporation of fuzziness in the spatial domain, i.e., in describing the geometry of regions, makes it possible to provide more meaningful results than by considering fuzziness in grey level alone. The effectiveness of the algorithms is demonstrated for different bandwidths of the membership function using a blurred chromosome image having a bimodal histogram and a noisy tank image having a unimodal histogram as input.

Key words: Image enhancement, tresholding, compactness, fuzzy sets, fuzziness, fuzzy compactness.

1. Introduction

The problem of grey level thresholding plays a key role in image processing and recognition. For example, in enhancing contrast in an image, we need to select proper threshold levels so that some suitable non-linear transformation can highlight a desirable set of pixel intensities compared to others. Similarly, in image segmentation one needs proper histogram thresholding whose objective is to establish boundaries in order to partition the image space (crisply) into meaningful regions.

When the regions in an image are ill-defined (i.e., fuzzy), it is natural and also appropriate to avoid committing ourselves to a specific segmentation by allowing the segments to be fuzzy subsets of the image. Fuzzy geometric properties (which are the generalization of those for ordinary regions) as defined by Rosenfeld [1–6] seem to provide a helpful tool for such analysis.

The present paper is an attempt to perform the above mentioned task automatically with the help of a compactness measure [4] which takes into account fuzziness in the spatial domain, i.e., in the geometry of the image regions. Besides this measure, we have also considered the ambiguity in grey level through the concepts of index of fuzziness [6], entropy [7] and index of nonfuzziness (crispness) [8]. These concepts were found by Pal [9–13] to provide objective measures for image enhancement, threshold selection, feature evaluation and seed point extraction.

The algorithms described here extract the fuzzy segmented version of an ill-defined image by minimizing the ambiguity in both the intensity and spatial domain. For making a nonfuzzy decision one may consider the cross-over point of the corresponding S function [14] as the threshold level. The nonfuzzy decisions corresponding to various algorithms are compared here when a blurred chromosome image and a noisy tank image are used as input.

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2. Measures of fuzziness in an image [8–10, 13]

An image X of size $M \times N$ and L levels can be considered as an array of fuzzy singletons, each having a value of membership denoting its degree of brightness relative to some brightness level l, $l = 0, 1, 2, \ldots, L-1$. In the notation of fuzzy sets, we may therefore write $X = \{\mu_X(x_{mn}) = \mu_{mn}/x_{mn};\ m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N\}$ where $\mu_X(x_{mn})$ or μ_{mn}/x_{mn} $(0 \le \mu_{mn} \le 1)$ denotes the grade of possessing some brightness property μ_{mn} (as defined in the next section) by the (m, n)th pixel intensity x_{mn}.

The index of fuzziness reflects the average amount of ambiguity (fuzziness) present in an image X by measuring the distance ('linear' and 'quadratic' corresponding to linear index of fuzziness and quadratic index of fuzziness) between its fuzzy property μ_X and the nearest two-level property $\mu_{\underline{X}}$; in other words, the distance between the gray tone image and its nearest two-tone version. The term 'entropy', on the other hand, uses Shannon's function but its meaning is quite different from classical entropy because no probabilistic concept is needed to define it. The index of nonfuzziness, as its name implies, measures the amount of nonfuzziness (crispness) in μ_X by computing its distance from its complement version. These quantities are defined below.

(a) Linear index of fuzziness

$$\nu_l(X) = \frac{2}{MN}\sum_m\sum_n |\mu_X(x_{mn}) - \mu_{\underline{X}}(x_{mn})| \tag{1a}$$

$$= \frac{2}{MN}\sum_m\sum_n \mu_{X \cap \overline{X}}(x_{mn}) \tag{1b}$$

$$= \frac{2}{MN}\sum_m\sum_n \min(\mu_X(x_{mn}), 1 - \mu_X(x_{mn})),$$

$$m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N,$$

where $\mu_{\underline{X}}(x_{mn})$ denotes the nearest two-level version of X such that

$$\mu_{\underline{X}}(x_{mn}) = 0 \quad \text{if } \mu_X(x_{mn}) \le 0.5, \tag{2a}$$

$$= 1 \quad \text{otherwise.} \tag{2b}$$

(b) Quadratic index of fuzziness

$$\nu_q(X) = \frac{2}{\sqrt{MN}}\left[\sum_m\sum_n [\mu_X(x_{mn}) - \mu_{\underline{X}}(x_{mn})]^2\right]^{0.5}, \tag{3}$$

$$m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N.$$

(c) Entropy

$$H(X) = \frac{1}{MN \ln 2}\sum_m\sum_n S_n(\mu_X(x_{mn})) \tag{4a}$$

with

$$S_n(\mu_X(x_{mn})) = -\mu_X(x_{mn})\ln \mu_X(x_{mn}) - (1 - \mu_X(x_{mn}))\ln(1 - \mu_X(x_{mn})),$$

$$m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N. \tag{4b}$$

(d) Index of nonfuzziness (crispness)

$$\eta(X) = \frac{1}{MN}\sum_m\sum_n |\mu_X(x_{mn}) - \mu_{\overline{X}}(x_{mn})|, \tag{5}$$

$\overline{X}$ is the complement of X,

$m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N.$

All these measures lie in $[0, 1]$ and have the following properties

$$I(X) = 0 \text{ (min)} \quad \text{for } \mu_X(x_{mn}) = 0 \text{ or } 1,\ \forall(m,n), \tag{6a}$$

$$I(X) = 1 \text{ (max)} \quad \text{for } \mu_X(x_{mn}) = 0.5,\ \forall(m,n), \tag{6b}$$

$$I(X) \ge I(X^*), \tag{6c}$$

$$I(X) = I(\overline{X}), \tag{6d}$$

where I stands for $\nu(X)$, $H(X)$ and $1 - \eta(X)$. X^* is the 'sharpened' or 'intensified' version of X such that

$$\mu_{X^*}(x_{mn}) \ge \mu_X(x_{mn}) \quad \text{if } \mu_X(x_{mn}) \ge 0.5, \tag{7a}$$

$$\le \mu_X(x_{mn}) \quad \text{if } \le 0.5. \tag{7b}$$

3. Fuzzy geometry of image subsets [1–5, 13]

Rosenfeld [1–5] extended the concepts of digital picture geometry to fuzzy subsets and generalized some of the standard geometric properties of and relationships among regions to fuzzy subsets. Among the extensions of the various properties, we only discuss here the area, perimeter and compactness of a fuzzy image subset, characterized by $\mu_X(x_{mn})$, which will be used in the following section for developing threshold selection algorithms. In defining the above mentioned parameters we replace $\mu_X(x_{mn})$ by μ for simplicity.

The area of μ is defined as

$$a(\mu) \triangleq \int \mu \tag{8}$$

where the integral is taken over any region outside which $\mu = 0$.

If μ is piecewise constant (for example, in a digital image) $a(\mu)$ is the weighted sum of the areas of the regions on which μ has constant values, weighted by these values.

For the piecewise constant case, the perimeter of μ is defined as

$$p(\mu) \triangleq \sum_{i,j}\sum_{k} |\mu_i - \mu_j|\,|A_{ijk}| \tag{9}$$
$$i,j = 1, 2, \ldots, r;\ i < j;\ k = 1, 2, \ldots, r_{ij}.$$

This is just the weighted sum of the length of the arcs A_{ijk} along which the i-th and j-th regions having constant μ values μ_i and μ_j respectively meet, weighted by the absolute difference of these values.

The compactness of μ is defined as

$$\text{comp}(\mu) \triangleq a(\mu)/p^2(\mu). \tag{10}$$

For crisp sets, this is largest for a disk, where it is equal to $1/4\pi$. For a fuzzy disk where μ depends only on the distance from the origin (center), it can be shown that

$$a(\mu)/p^2(\mu) \geq 1/4\pi. \tag{11}$$

In other words, of all possible fuzzy disks, the compactness is smallest for its crisp version. For this reason, in this paper we will use minimization (rather than maximization) of fuzzy compactness as a criterion for image enhancement and threshold selection.

4. Threshold selection

A. Minimizing fuzziness [10, 13]

Let us consider for example, the minimization of $v_l(X)$. It is seen from equations (2) that the nearest ordinary plane μ_X (which represents the closest two-tone version of the grey tone image X) is dependent on the position of the cross-over point, i.e., the 0.5 value of μ_X. Therefore a proper selection of the cross-over point may be made which will result in a minimum value of $v(X)$ only when the cross-over point corresponds to the appropriate boundary between regions (clusters) in X.

This can be explained further as follows. Suppose we consider the standard S-function (Figure 1) [14]

$$\begin{aligned}
\mu_X(x_{mn}) &= S(x_{mn}; a, b, c) \\
&= 0, && x_{mn} \leq a, && \text{(12a)} \\
&= 2[(x_{mn} - a)/(c-a)]^2, && a \leq x_{mn} \leq b, && \text{(12b)} \\
&= 1 - 2[(x_{mn} - c)/(c-a)]^2, && b \leq x_{mn} \leq c, && \text{(12c)} \\
&= 1, && x_{mn} \geq c, && \text{(12d)}
\end{aligned}$$

with cross-over point $b = (a + c)/2$ and bandwidth

$$\Delta b = b - a = c - b$$

for obtaining $\mu_X(x_{mn})$ or μ_{mn} (representing the degree of brightness of each pixel) from the given x_{mn} of the image X. Then for a cross-over point selected at, say, $b = l_i$ we have $\mu_X(l_i) = 0.5$ and μ_{mn} would take on values > 0.5 and < 0.5 corresponding to $x_{mn} > l_i$ and $< l_i$; which implies allocation of the grey levels into two ranges. The term $v(X)$ then measures the average ambiguity in X by computing $\mu_{X \cap \bar{X}}(x_{mn})$ in such a way that the contribution of the levels towards $v(X)$ comes mostly from those near l_i and decreases as we move away from l_i.

Therefore, modification of the cross-over point will result in different segmented images with vary-

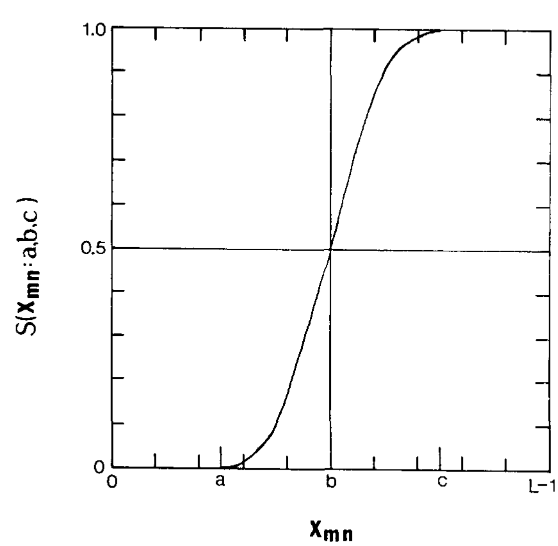

Figure 1. *S* function.

ing $v(X)$. When b corresponds to the appropriate boundary (threshold) between two regions, there will be a minimum number of pixel intensities in X having $\mu_{mn} \simeq 0.5$ (resulting in $v \simeq 1$) and a maximum number of pixel intensities having $\mu_{mn} \simeq 0$ or 1 (resulting in $v \simeq 0$) thus contributing least towards $v(X)$. This optimum (minimum) value of fuzziness would be greater for any other selection of the cross-over point.

Method of computation (Algorithm 1)

Given an $M \times N$ image with minimum and maximum grey levels $l_{\min}$ and $l_{\max}$:

Step 1. Construct the 'bright image' membership μ_X, where

$$\mu_X(l) = S(l; a, l_i, c), \quad l_{\min} \leq l, l_i \leq l_{\max}, \tag{13}$$

using equation (12) with cross-over point $b = l_i$ and a particular bandwidth $\Delta b = c - l_i = l_i - a$.

Step 2. Compute the amount of fuzziness in μ_X corresponding to $b = l_i$ with

$$v(X) \mid l_i = \frac{2}{MN} \sum_l \min\{S(l; a, l_i, c), 1 - S(l; a, l_i, c)\} h(l) \tag{14a}$$

$$= \frac{2}{MN} \sum_l T_i(l) h(l) \tag{14b}$$

where

$$T_i(l) = \min\{S(l; a, l_i, c), 1 - S(l; a, l_i, c)\} \tag{14c}$$

and $h(l)$ denotes the number of occurrences of the level l.

Step 3. Vary l_i from $l_{\min}$ to $l_{\max}$ and select $l_i = l_c$, say, for which $v(X)$ is a minimum.

l_c is thus the cross-over point of $\mu_X(x_{mn})$ having minimum ambiguity (i.e., for which μ_X has minimum distance from its closest two-tone version). μ_{mn} can be regarded as a *fuzzy segmented version* of the image, with $\mu_{mn} < 0.5$ and > 0.5 corresponding to regions $[l_{\min}, l_c - 1]$ and $[l_c, l_{\max}]$.

For the purpose of nonfuzzy segmentation, one can consider the level l_c as the threshold between background and object, or the boundary of the object region. This can further be verified from equation (14) which shows that the minimum value of $v(X)$ would always correspond to the valley region of the histogram having minimum number of occurrences.

Variation of bandwidth (Δb)

Let us call $T_i(l)$ (equation 14(c)) a Triangular Window function centered at l_i with bandwidth Δb. As Δb decreases, μ_X would have more intensified contrast around the cross-over point resulting in decrease of ambiguity in μ_X. As a result, the possibility of detecting some undesirable thresholds (spurious minima in the histogram) increases because of the smaller width of the $T_i(l)$ function.

On the other hand, increase of Δb results in a higher value of fuzziness and thus leads toward the possibility of losing some of the weak minima.

The application of this technique to both bimodal and multimodal images with various T_i functions based on $v_l(X)$, $v_q(X)$, $H(X)$ and $\eta(X)$ is demonstrated in [10, 13].

B. Minimizing compactness

In the previous discussion of threshold selection we considered fuzziness in the grey levels of an image. In this section we take fuzziness in the spatial domain into consideration by using the compactness measure for selecting nonfuzzy thresholds.

It is seen from Section 3 that both the perimeter and area of a fuzzy segmented image depend on the membership value, denoting the degree of brightness, say, of each region. It is further to be noted that the compactness of a fuzzy region decreases as its μ value increases and it is smallest for a crisp one. We will now define two algorithms to show how the above mentioned concept can be utilized for selecting a threshold between two regions (say, the background and a single object) in a bimodal image X.

As in the case of the previous algorithm, we construct μ_{mn} with different S functions having constant

Δb value and select the cross-over pointof the μ_X as the boundary of the object for which comp(μ) is a minimum.

Method of computation (Algorithm 2)

Given an $M \times N$ image with minimum and maximum grey levels $l_{\min}$ and $l_{\max}$:

Step 1. Construct 'bright' image μ_X as in Step 1 of Algorithm 1.

Step 2. Compute the area and perimeter of μ_X corresponding to $b = l_i$ with

$$a(\mu) \mid l_i = \sum_m \sum_n \mu_{mn} = \sum_l S(l;\, a,\, l_i,\, c) h(l), \tag{15}$$
$$m = 1, 2, \ldots, M;\; n = 1, 2, \ldots, N;$$
$$l_{\min} \le l,\, l_i \le l_{\max}$$

and

$$p(\mu) \mid l_i = \sum_{m=1}^{M} \sum_{n=1}^{N-1} \left| \mu_{mn} - \mu_{m,n+1} \right| + \sum_{n=1}^{N} \sum_{m=1}^{M-1} \left| \mu_{mn} - \mu_{m+1,n} \right| \tag{16}$$

(excluding the frame of the image).

For example, consider the 4×4 μ_{mn} array

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & \alpha & \beta & 0 \\ 0 & 0 & \beta & \gamma \\ 0 & \delta & 0 & 0 \end{matrix}, \quad 1 \ge \alpha, \beta, \gamma, \delta > 0.$$

Here, $a(\mu) = \alpha + 2\beta + \gamma + \delta$
and

$$p(\mu) = [\alpha + |\beta - \alpha| + \beta + \beta + |\gamma - \beta| + \delta + \delta] + [\alpha + \alpha + \delta + \beta + 0 + \beta + \gamma + \gamma].$$

Step 3. Compute the compactness of μ_X corresponding to $b = l_i$ with

$$\mathrm{comp}(\mu) \mid l_i = \frac{a(\mu) \mid l_i}{p^2(\mu) \mid l_i}. \tag{17}$$

Step 4. Vary l_i from $l_{\min}$ to $l_{\max}$ and select that $l_i = l_c$, say, for which comp(μ) is minimum

The level l_c therefore denotes the cross-over point of the fuzzy image plane μ_{mn} which is least compact (or most crisp). The μ_{mn} so obtained can therefore be viewed as a *fuzzy segmented version* of the image X.

Like the previous algorithm, one can consider l_c as the threshold for making a nonfuzzy decision on classifying/segmenting the image into regions.

Method of computation (Algorithm 3)

Here we approximate the definitions of area and compactness of μ_X by considering that μ_X has only two values corresponding to the background and object regions. The μ value for the background is assumed to be zero, whereas the μ-value of the object region is monotonically increasing with increase in threshold level. Therefore, by varying the threshold, one can have different segmented versions of the object region. Each segmented version thresholded at l_t has its area and perimeter computed as follows:

$$a(\mu_t) = a \cdot \mu_t \tag{18a}$$
$$= \mu_t \sum_l h(l), \quad l_t \le l \le l_{\max}, \tag{18b}$$

where a denotes the area of the region on which $\mu = \mu_t$ (constant), i.e., the number of pixels having grey level greater than or equal to l_t and

$$p(\mu_t) = \mu_t \cdot p \tag{18c}$$

where p denotes the length of the arcs along which the regions having $\mu = \mu_t$ and $\mu = 0$ meet, or, in other words, the perimeter of the region on which $\mu = \mu_t$ (constant).

For the example considered in Algorithm 2, the values of $a(\mu_t)$ and $p(\mu_t)$ for $\alpha = \beta = \gamma = \delta = \mu_t$ will be $5\mu_t$ and $12\mu_t$ respectively.

The algorithm for selecting the boundary of a single-object region from an $M \times N$ dimensional image may therefore be stated as follows:

Step 1. Construct the 'bright' image μ_X using

$$\mu_X(l) = S(l;\, a,\, b,\, c) \tag{19}$$

with $a = l_{\min}$, $c = l_{\max}$ and $b = (a + c)/2$.

Step 2. Generate a segmented version putting

$$\mu = 0 \quad \text{for } \mu < \mu_t, \tag{20a}$$

$$= \mu_t \quad \text{for } \mu \geq \mu_t, \tag{20b}$$

where μ_t is the value of $\mu_X(l_t)$ obtained in Step 1.

Step 3. Compute the compactness of the segmented version thresholded at l_t:

$$\text{comp}(\mu_t) = \frac{a \cdot \mu_t}{p^2 \cdot \mu_t^2} = \frac{a}{p^2 \cdot \mu_t}. \tag{21}$$

Step 4. Vary l_t in $(l_{\min}, l_{\max})$ and hence μ_t in $(0,1)$ and select the level as boundary of the object for which equation (21) attains its minimum.

It should be noted here that after approximation of the area and perimeter of μ_{mn}, the compactness measure (equation (21)) reduces to $1/\mu_t$ times the crisp compactness of the object region. Unlike Algorithms 1 and 2, here μ_X is kept fixed throughout the process and the output of the algorithm is a nonfuzzy segemted version of X determined by l_t.

C. *Minimizing the product (**Algorithm 4**)*

Algorithms 1–3 minimize either the amount of fuzziness or the compactness of an image X. We can combine these measures and compute the product of fuzziness and compactness, and determine the level for which the product becomes a minimum. In other words, we compute

$$\theta_{l_i} = v(X)\,|_{l_i} \cdot \text{comp}(\mu)\,|_{l_i} \tag{22}$$
(using equations (14) and (17))

$$\text{or } \theta_{l_t} = v(X)\,|_{l_t} \cdot \text{comp}(\mu_t) \tag{23}$$
(using equations (14) and (21))

at each value of l_i (or l_t), $l_{\min} < l_i, l_t < l_{\max}$, and select $l_i = l_c$, say, as threshold for which equation (22) (or (23)) is a minimum. The corresponding μ_{mn} represents the *fuzzy segmented version* of the image as far as minimization of its fuzziness in grey level and the spatial domain is concerned.

It should be mentioned here that although we considered the linear index of fuzziness in Algorithms 1 and 4, one can also consider the other measures, namely $v_q(X)$, $H(X)$ and $\eta(X)$, for computing the total amount of fuzziness in μ_{mn}.

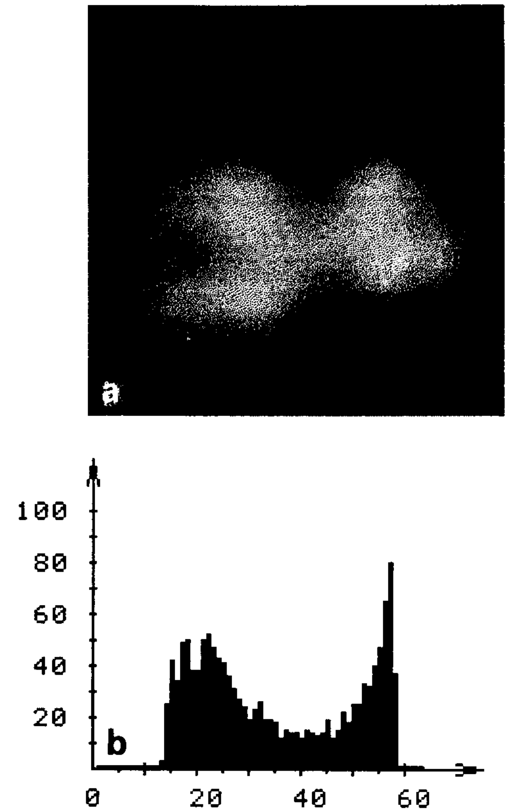

Figure 2. (a) Chromosome image; (b) Histogram.

5. Implementation and results

Figure 2a shows a 64 × 64, 64 level image of a blurred chromosome with $l_{\min} = 12$ and $l_{\max} = 59$. Figure 2b shows its bimodal histogram.

The different minima obtained using Algorithms 1–4 for $\Delta b = 2$, 4, 8, 16 are given in Table 1. The enhanced version of the chromosome corresponding to these thresholds (minima) are shown in Figures 3 to 8 only for $\Delta b = 4$, 8 and 16. In each of Figures 3–5, (a), (b) and (c) correspond to Algorithm 1, Algorithm 2 and equation (22) of Algorithm 4. Similarly, in Figures 6–7, (a), (b) and (c) correspond to Algorithm 1, Algorithm 3 and equation (23) of Algorithm 4.

It is seen that the compactness measure usually results in more minima as compared to index of fuzziness. The index of fuzziness (Algorithm 1) basically sharpens the histogram and it detects a single threshold in the valley region of the histogram for $\Delta b = 4$, 8 and 16. At $\Delta b = 2$, the algorithm as expected results in some undesirable thresholds cor-

Table 1
Minima for chromosome image

	$\Delta b=2$	$\Delta b=4$	$\Delta b=8$	$\Delta b=16$
$\nu(X)$ (Algorithm 1)	19, 30, 40*, 46	40	40	46
comp(μ) Algorithm 2)	18, 24, 31, 56*	29, 54*	31, 47*	34
comp(μ) (Algorithm 3)	33, 48*	33, 48*	33, 48*	33, 48*
Product (eqn. (22) of Algorithm 4)	19, 31, 40, 42*, 46	40	41	41
Product (eqn. (23) of Algorithm 4)	40, 42, 44, 46*, 53	42*, 45, 53	42*, 45, 48	33, 48*

* Denotes global minimum.
Algorithm 3 does not involve variation of Δb.

responding to weak minima of the histogram. This conforms to the earlier investigation [10]. Algorithms 2 and 3 based on the compactness measure, on the other hand, detect a higher-valued threshold (global minimum) which results in better segmentation (or enhancement) of the chromosome as far as its shape is concerned.

The advantage of the compactness measures over the index value is that they take fuzziness in the spa-

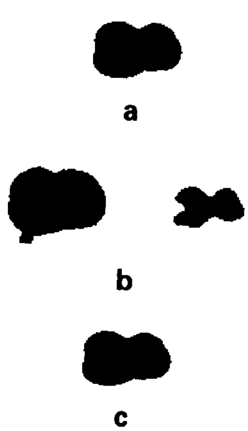

Figure 3. Enhanced/thresholded versions of chromosome for $\Delta b = 4$. (a) Algorithm 1 ($l_c = 40$); (b) Algorithm 2 ($l_c = 29, 54$); (c) Equation (22) ($l_c = 40$).

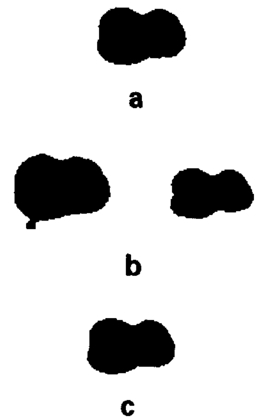

Figure 4. Enhanced/thresholded versions of chromosome for $\Delta b = 8$. (a) Algorithm 1 ($l_c = 40$); (b) Algorithm 2 ($l_c = 31, 47$); (c) Equation (22) ($l_c = 41$).

Figure 5. Enhanced/thresholded versions of chromosome for $\Delta b = 16$. (a) Algorithm 1 ($l_c = 46$); (b) Algorithm 2 ($l_c = 34$); (c) Equation (22) ($l_c = 41$).

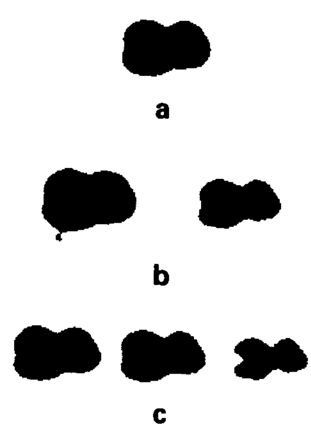

Figure 6. Enhanced/thresholded versions of chromosome for $\Delta b = 4$. (a) Algorithm 1 ($l_c = 40$); (b) Algorithm 3 ($l_c = 33, 48$); (c) Equation (23) ($l_c = 42, 45, 53$).

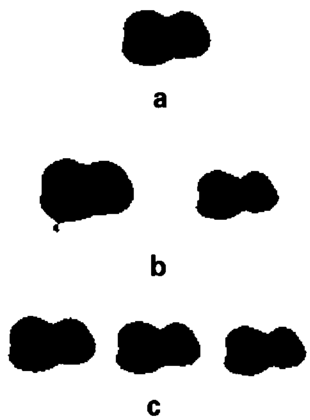

Figure 7. Enhanced/thresholded versions of chromosome for $\Delta b = 8$. (a) Algorithm 1 ($l_c = 40$); (b) Algorithm 3 ($l_c = 33, 48$); (c) Equation (23) ($l_c = 42, 45, 48$).

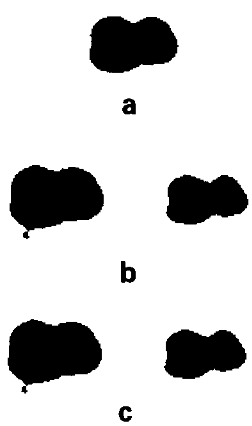

Figure 8. Enhanced/thresholded versions of chromosome for $\Delta b = 16$. (a) Algorithm 1 ($l_c = 46$); (b) Algorithm 3 ($l_c = 33, 48$); (c) Equation (23) ($l_c = 33, 48$).

tial domain (i.e., the geometry of the object) into consideration in extracting thresholds. The index value, on the other hand, incorporates fuzziness only in grey level. It should further be noted for Algorithm 2 that as Δb increases, the number of and the separation between minima also decrease.

It is interesting to note that multiplying $v(X)$ by $\text{comp}(\mu_t)$, i.e., equation (23), produces at least as many thresholds as are generated by the individual measures. But this is not the case for equation (22) where the number of thresholds is (except for $\Delta b = 2$) equal to or less than the numbers for the individual measures.

The above observations can be explained as follows. As mentioned before, $v(X)$ basically sharpens the histogram. Therefore as l_i increases, it first increases until it reaches a maximum, and then decreases until a minimum (threshold) is attained. After this it follows the same pattern for the other mode of the histogram. The compactness measure, on the other hand, first starts decreasing until it reaches a minimum, then increases for a while, and then starts decreasing again.

It is further seen from our results that the variation of compactness in Algorithm 3 plays a more dominant role than the variation of index value in Algorithm 1 in detecting minima. The case is reversed for the combination of Algorithm 1 and Algorithm 2, where the product is influenced more by the index value. As a result, the threshold obtained by equation (22) is found to be within the range of threshold values obtained by the individual measures. Equation (23), on the other hand, is able to create a higher-valued (or at least equal) threshold which results in better object enhancement than those of the individual measures.

Figures (9(a) and 9(b) show a noisy image of a tank and its unimodal histogram, having $l_{min} = 14$,

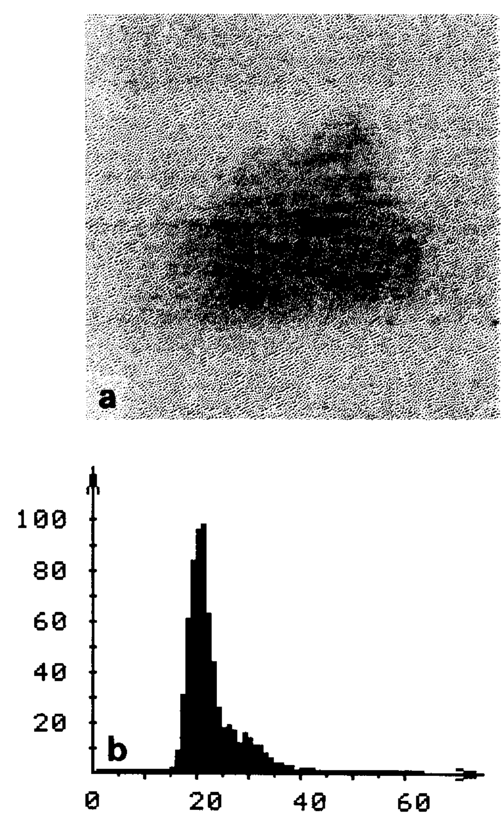

Figure 9. (a) Tank image; (b) Histogram.

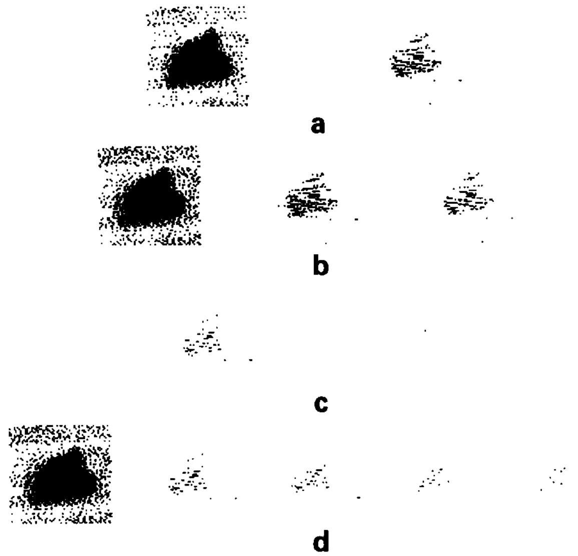

Figure 10. Enhanced/thresholded versions of tank for $\Delta b = 4$. (a) Algorithm 2 ($l_c = 23, 34$); (b) Algorithm 3 ($l_c = 22, 33, 36$); (c) Equation (22) ($l_c = 40, 49$); (d) Equation (23) ($l_c = 22, 40, 42, 44, 46$).

$l_{max} = 50$. The minima obtained by the different algorithms for $\Delta b = 2$, 4, 8 and 16 are given in Table 2. The corresponding enhanced versions for $\Delta b = 4$, 8 and 16 are shown in Figures 10–12 for various combinations of algorithms.

As expected, the index of fuzziness alone was not able to detect a threshold for the tank image because of its unimodal histogram. The compactness measure, on the other hand, does give good thresholds. As in the case of the chromosome image, equation (23) yields at least as many thresholds as

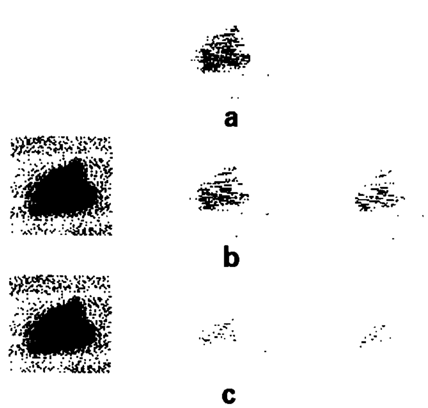

Figure 11. Enhanced/thresholded versions of tank for $\Delta b = 8$. (a) Algorithm 2 ($l_c = 31$); (b) Algorithm 3 ($l_c = 22, 33, 36$); (c) Equation (23) ($l_c = 22, 42, 44$).

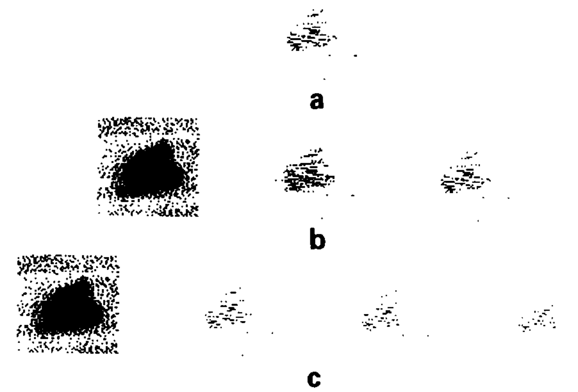

Figure 12. Enhanced/thresholded versions of tank for $\Delta b = 16$. (a) Algorithm 2 ($l_c = 36$); (b) Algorithm 3 ($l_c = 22, 33, 36$); (c) Equation (23) ($l_c = 22, 38, 40, 42$).

are generated by the compactness measure. Similarly (except for $\Delta b = 2$) equation (22) yields at most as many thresholds as the compactness measure.

6. Conclusions

Algorithms based on compactness measures of fuzzy sets are developed and used to determine thresholds (both fuzzy and nonfuzzy) of an ill-defined image (or the enhanced version of a fuzzy ob-

Table 2
Minima for tank image

	$\Delta b=2$	$\Delta b=4$	$\Delta b=8$	$\Delta b=16$
$v(X)$ (Algorithm 1)	–	–	–	–
comp(μ) (Algorithm 2)	21*, 33	23*, 34	31	36
comp(μ) (Algorithm 3)	22, 33*, 36	22, 33*, 36	22, 33*, 36	22, 33*, 36
Product (eqn. (22) of Algorithm 4)	24, 39*, 43, 46, 49	40, 49*	–	–
Product (eqn. (23) of Algorithm 4)	24, 40, 42, 46*	22, 40, 42*, 44, 46	22, 42*, 44	22, 38, 40, 42*

* Denotes global minimum.
Algorithm 3 does not involve variation of Δb.

ject region) without referring to its histogram. The enhanced chromosome images obtained from the global minima of the measures are found to be better than those obtained on the basis of minimizing fuzziness in grey level, as far as the shape of the chromosome is concerned. Consideration of fuzziness in the spatial domain, i.e., in the geometry of the object region, provides more information by making it possible to extract more than a single thresholded version of an object. Similarly in the case of the unimodal (noisy) tank image, the compactness measure is able to determine some suitable thresholds but the index parameter is not. Furthermore, optimization of both compactness and fuzziness usually allows better selection of thresholded/ enhanced versions.

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