Image analysis with subpixel precision - The Coverage model

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Models and methods for precise image analysis

Few words about the project:

- **Info**
  - http://imft.ftn.uns.ac.rs/~natasa/CoverageModelCourse
  - Joakim and Nataša are sitting in Room 2144

- **Goals**
  - To initiate concrete research tracks which include practical application of the framework of the coverage model to already existing research projects at CBA.
  - To further develop the framework through cooperation.
  - Collaborations initiated will be intensively carried on during the visit, along with the course, which will serve as a discussion platform/idea incubator.
  - The collaborating projects will be continued, with an aim to be brought to some results and conclusions (hopefully, some publications) during the expected additional visit the following year.

Motivation and context

The task of **Image Analysis** is to **extract relevant information from images**.

- **Numerical descriptors**, such as area, perimeter, and moments of objects are often of interest, for the tasks of shape analysis, classification, etc.

The standard image analysis task (and its solution)

- Sample preparation and Imaging
- Pre-processing (optional)
- Segmentation
- Usually crisp
- Feature extraction
  - Discrete representation problematic
- Classification

Crisp discrete object representations, especially at low spatial resolutions, put strong limitations to the precision of estimated features.
Motivation - feature extraction

Unrealistically low resolution?
- Low resolution will always be a challenge; the more powerful imaging devices, the smaller objects are of interest!
- Partial Volume Effect in high resolution 3D images; consistent small displacement of object boundaries leads to significant errors of feature estimates.

Approach based on fuzzy sets

A fuzzy set of a reference set is a set of ordered pairs

\[ F = \{ (x, \mu_F(x)) \mid x \in X \} , \]

where the membership function \( \mu_F : X \to [0, 1] \) indicates, for each element \( x \in X \), to what extent it belongs to the fuzzy set \( F \).

Observations:
- A fuzzy set is defined/identified by its membership function
- A crisp set is a special case of fuzzy set, where membership function takes only two values, 0 and 1.

The Fuzzy Approach

- Do not throw away information by making crisp decisions.
- The more nuanced view of a fuzzy approach allows preservation of more information.
- A representation based on fuzzy sets can provide numerical descriptors with higher precision than what can be achieved from a crisp representation.

An example of a fuzzy segmented image

Breast density, as measured from the volume of dense tissue in the breast is considered to indicate a risk factor for breast cancer.

A digitized X-ray mammogram, the fuzzy connectivity scene of a dense (fibroglandular) region (as opposed to fatty regions), and a corresponding crisp region.

The Fuzzy Approach

A standard image analysis task and its fuzzy solution

- Sample preparation and Imaging
- Pre-processing (optional)
- Segmentation (a lot of freedom)
  - Fuzzy segmentation
  - May be difficult to interpret the results; different meanings of memberships
- Feature extraction
- Classification, statistical evaluation, …

New entries in the “standard chain of tasks” required:
- New segmentation methods that result in fuzzy representations;
- New analysis methods that can be applied to fuzzy object representations;
- New ways of interpreting and understanding the obtained results.

Important observations

- The original grey levels are, in general, not directly used as fuzzy representations.
- Appropriately designed fuzzy segmentation methods combine original grey levels together with several other types of information to define membership value of a pixel to an object.
- Meaning of grey levels in a fuzzy segmented image is always a consequence of many criteria, many of them only implicitly present.
- Instead of dependence on the properties of imaging devices, we are dependent on criteria used for fuzzy segmentation.
- Interpretation of results can become rather difficult.
The Coverage Model

- Keep good sides of fuzzy; stay close to the digital image, high information content, soft boundaries, robustness.
- Restrict to one single meaning of memberships; clear unique interpretation, enabling stronger theoretical results.

The Coverage Model

- Let the (membership) value of an image element be equal to its relative coverage by the image object.
- A representation close to the original (discrete) image data.
- Based on very weak assumptions about the imaged objects.
- Utilizing the coverage information, significant improvement in precision of extracted feature values can be reached.

Properties of coverage representations

Intuitively, a coverage representation of a crisp object, with a well defined continuous border, is characterized by the presence of homogeneous connected regions of “pure” pixels, completely covered by either object or background separated by thin layers of “mixed” pixels, i.e., those partially covered by both object and background.

If a crisp continuous set has a reasonably smooth boundary and is represented at a high enough resolution, then the fuzzy border of its digital coverage representation is not more than one pixel thick.
Feature extraction revisited