Introduction

Classical logic: A brief overview

- **Logic** is the study of methods and principles of **reasoning** in all its possible forms.
- **Propositions** - statements that are required to be **true** or **false**.
- The **truth value** of a proposition is the opposite of the truth value of its **negation**.
- Instead of propositions, we use **logic variables**. Logic variable may assess one of the two truth values, if it is substituted by a particular proposition.
- **Propositional logic** studies the rules by which new logic variables can be produced from some given logic variables. The internal structure of the propositions “behind” the variables does not matter!

- Classical logic - a very brief overview  
  - Chapter 8.1
- Multivalued logic  
  - Chapter 8.2
Logic function assigns a truth value to a combination of truth values of its variables:

\[ f : \{\text{true, false}\}^n \rightarrow \{\text{true, false}\} \]

2^n choices of n arguments \(\rightarrow\) 2^{2^n} logic functions of n variables.

• Observe, e.g.:

\[
\omega_{14}(v_1, v_2) = \omega_{15}(\omega_6(v_1, v_2), v_2) \\
\omega_{10}(v_1, v_2) = \omega_9(\omega_{14}(v_1, v_2), \omega_{12}(v_1, v_2))
\]

• A task: Express all the logic functions of n variables by using only a small number of simple logic functions, preferably of one or two variables.

• Such a set is a complete set of logic primitives.

• Examples:

\{\text{negation, conjunction, disjunction}\} = \{\omega_6, \omega_9, \omega_{15}\},
\{\text{negation, implication}\} = \{\omega_6, \omega_{14}\}.

### Definition

1. If \(v\) is a logic variable, then \(v\) and \(\bar{v}\) are logic formulae;

2. If \(v_1\) and \(v_2\) are logic formulae, then \(v_1 \land v_2\) and \(v_1 \lor v_2\) are also logic formulae;

3. Logic formulae are only those defined (obtained) by the two previous rules.
Classical logic: A brief overview

Logic formulae

Each logic formula generates a unique logic function. Different logic formulae may generate the same logic function. Such are called equivalent.

Examples:

\[(v_1 \Rightarrow v_2) \iff (\neg v_1 \lor v_2)\]
\[(v_1 \Leftrightarrow v_2) \iff ((v_1 \Rightarrow v_2) \land (v_1 \Leftarrow v_2))\]

Tautology is (any) logic formula that corresponds to a logic function one. Contradiction is (any) logic formula that corresponds to a logic function zero.

Predicate logic

There are situations when the internal structure of propositions cannot be ignored in deductive reasoning.

Propositions are, in general, of the form

\[x \text{ is } P\]

where \(x\) is a symbol of a subject and \(P\) is a predicate that characterizes a property.

\(x\) is any element of universal set \(X\), while \(P\) is a function on \(X\), which for each value of \(x\) forms a proposition.

\(P(x)\) is called predicate; it becomes true or false for any particular value of \(x\).

Inference rules

Inference rules are tautologies used for making deductive inferences.

Examples:

\[(a \land (a \Rightarrow b)) \Rightarrow b\] modus ponens
\[(\neg b \land (a \Rightarrow b)) \Rightarrow \neg a\] modus tollens
\[(a \Rightarrow b) \land (b \Rightarrow c) \Rightarrow (a \Rightarrow c)\] hypothetical syllogism

Predicate logic-extensions

\(- n\text{-ary predicates } P(x_1, x_2, \ldots, x_n)\)

Quantification of applicability of a predicate with respect to the domain of its variables

Existential quantification: \((\exists x)P(x)\)

Universal quantification: \((\forall x)P(x)\)

It holds:

\[\left(\exists x\right) P(x) = \bigvee_{x \in X} P(x)\]
\[\left(\forall x\right) P(x) = \bigwedge_{x \in X} P(x)\]
Multivalued Logics

Three-valued logic

- Third truth value is allowed:
  - truth: 1
  - false: 0
  - intermediate: \( \frac{1}{2} \).
- While it is accepted to have \( \bar{p} = 1 - p \), the definitions of other primitives differ in different three-valued logics.
- For the best known three-valued logics (Łukasiewicz, Bochvar, Kleene, Heyting, Reichenbach), primitives coincide with two valued counterparts for the variables having values 0 or 1 (see Table 8.4, p.218).
- None of the mentioned logics satisfies law of excluded middle, or law of contradiction.
- quasi-tautology is a logic formula that never assumes truth value 0;
- quasi-contradiction is a logic formula that never assumes truth value 1.

Łukasiewicz \( n \)-valued logic

The set of truth values is

\[
T_n = \{0 = \frac{0}{n-1}, \frac{1}{n-1}, \frac{2}{n-1}, \ldots, \frac{n-2}{n-1}, \frac{n-1}{n-1} = 1\}.
\]

Truth values are interpreted as \textit{degrees of truth}.

Primitives in \( n \)-valued logics of Łukasiewicz, denoted by \( L_n \), are:

\[
\bar{p} = 1 - p \quad p \land q = \min[p, q] \quad p \lor q = \max[p, q] \quad p \Rightarrow q = \min[1, 1 + q - p] \quad p \iff q = 1 - |p - q|
\]

Fuzzy logic

- Fuzzy propositions
- Linguistic hedges
- Fuzzy quantifiers

Chapter 8.3
Chapter 8.5
Chapter 8.4
Fuzzy propositions

The range of truth values of fuzzy propositions is not only \( \{0, 1\} \), but \( [0, 1] \).
The truth of a fuzzy proposition is a matter of degree.

Classification of fuzzy propositions:

- **Unconditional and unqualified** propositions
  “The temperature is high.”

- **Unconditional and qualified** propositions
  “The temperature is high is very true.”

- **Conditional and unqualified** propositions
  “If the temperature is high, then it is hot.”

- **Conditional and qualified** propositions
  “If the temperature is high, then it is hot is true.”

Linguistic hedges (modifiers)

For a given predicate \( F \) on \( X \) and a given linguistic hedge \( H \), a new (modified) fuzzy predicate \( HF \) is defined as:

\[
HF(x) = h(F(x)), \quad \text{for all } x \in X.
\]

A modifier \( h \) is a unary operation \( h: [0, 1] \rightarrow [0, 1] \) such that:

- \( h(0) = 0 \) and \( h(1) = 1 \):
- \( h \) is a continuous function;
- If \( h(a) < a \) for all \( a \in [0, 1] \), (i.e., if \( h \) is **strong**), then \( h^{-1}(a) > a \) for all \( a \in [0, 1] \), (i.e., then \( h^{-1} \) is **weak**).
- A composition of modifiers is also a modifier.

**Strong** modifier reduces the truth value of a proposition.
**Weak** modifier increases the truth value of a proposition (by weakening the proposition).
An identity modifier is a function \( h(a) = a \).

Linguistic hedges

- **Linguistic hedges** are linguistic terms by which other linguistic terms are modified.
  “Tina is young is true.”
  “Tina is very young is true.”
  “Tina is very young is very true.”

- Fuzzy predicates and fuzzy truth values can be modified.
  Crisp predicates cannot be modified.
- Examples of hedges: very, fairly, extremely.

Modifiers

One commonly used class of modifiers is

\[
h_{\alpha}(a) = a^\alpha, \quad \text{for } \alpha \in \mathbb{R}^+ \text{ and } a \in [0, 1].
\]

For \( \alpha < 1 \), \( h_{\alpha} \) is a weak modifier.
**Example:** \( H: \text{fairly} \leftrightarrow h(a) = \sqrt{a} \).

For \( \alpha > 1 \), \( h_{\alpha} \) is a strong modifier.
**Example:** \( H: \text{very} \leftrightarrow h(a) = a^2 \).

\( h_1 \) is the identity modifier.
Fuzzy quantifiers

To determine the truth value of a quantified proposition, we need to know

1. “how many” students in the group are high-fluent
   i.e., cardinality of a fuzzy set High-fluent

2. “how much” is that value about 3
   i.e., membership of the obtained value
to the fuzzy set About 3

or

1. “how many” students in the group are high-fluent,
   relatively to the size of the group
   i.e., cardinality of a fuzzy set High-fluent
   divided by the size of the group

2. “how much” is that value almost all
   i.e., membership of the obtained value
to the fuzzy set Almost all.

Examples:

\[ p: \text{There are } 3 \text{ high-fluent students in the group.} \]
\[ q: \text{Almost all students in the group are high-fluent.} \]

Fuzzy quantifiers

• **Absolute** quantifiers:
  “about 10”; “much more than 100”, ...

• **Relative** quantifiers:
  “almost all”, “about half”, ...

Group = \{ Adam, Bob, Cathy, David, Eve \}. Fluency is represented by the value from the interval \([0, 100]\). Fuzzy set \( F \) represents “High fluency” on \([0,100]\). Fuzzy set \( Q \) represents fuzzy quantifier “about 3”.

\[ E = 0/\text{Adam} + 0/\text{Bob} + 0.75/\text{Cathy} + 1/\text{David} + 0.5/\text{Eve} \]

is a fuzzy set “High fluency” on the domain Group.

\[ |E| = 2.25 \]
\[ T(p) = Q(|E|) = Q(2.25) = 0.625. \]
\[ \frac{|E|}{|\text{Group}|} = 0.45 \]
\[ T(q) = Q_l(0.45) = 0. \]
Fuzzy propositions

Unconditional and unqualified propositions

The canonical form

\[ p : \nu \text{ is } F \]

\( \nu \) is a variable on some universal set \( V \)
\( F \) is a fuzzy set on \( V \) that represents a fuzzy predicate (e.g., low, tall, young, expensive...)

The degree of truth of \( p \) is

\[ T(p) = F(\nu), \quad \text{for } \nu \in \nu. \]

\( T \) is a fuzzy set on \( V \). Its membership function is derived from the membership function of a fuzzy predicate \( F \).

The role of a function \( T \) is to connect fuzzy sets and fuzzy propositions.

In case of unconditional and unqualified propositions, the identity function is used.

Fuzzy propositions

Unconditional and qualified propositions

The canonical form

\[ p : \nu \text{ is } F \text{ is } S \quad \text{(truth qualified proposition)} \]

where \( \nu \) is a variable on some universal set \( V \),
\( F \) is a fuzzy set on \( V \) that represents a fuzzy predicate,
and \( S \) is a fuzzy truth qualifier.

To calculate the degree of truth \( T(p) \) of the proposition \( p \), we use:

\[ T(p) = S(F(\nu)) \]

An illustration:

Note: The proposition can be expressed as "\( \nu \text{ is } F \text{ is true} \)."

Fuzzy propositions

Unconditional and qualified propositions

An illustration:

\[ p: \text{ "Tina is young is very true". } \text{Tina is 26.} \]

\( \text{Young}(26) = 0.87 \text{, and } \text{VeryTrue}(0.87) = 0.76 \)

\[ T(p) = 0.76. \]
Fuzzy propositions
Conditional and unqualified propositions

The canonical form

\[ p : \text{If } X \text{ is } A, \text{ then } Y \text{ is } B, \]

where \( X, Y \) are variables on \( X, Y \) respectively, and \( A, B \) are fuzzy sets on \( X, Y \) respectively.

Alternative form:

\[ \langle X, Y \rangle \text{ is } R \]

where \( R(x, y) = J(A(x), B(x)) \) is a fuzzy set on \( X \times Y \) representing a suitable fuzzy implication.

Fuzzy implications

- Fuzzy implications
- Selection of fuzzy implications

Fuzzy propositions
Conditional and qualified propositions

The canonical form

\[ p : \text{If } X \text{ is } A, \text{ then } Y \text{ is } B \text{ is } S \]

where \( X, Y \) are variables on \( X, Y \) respectively, \( A, B \) are fuzzy sets on \( X, Y \) respectively, and \( S \) is a truth qualifier.

A fuzzy implication \( J \) of two fuzzy propositions \( p \) and \( q \) is a function of the form

\[ J : [0, 1] \times [0, 1] \rightarrow [0, 1], \]

which for any truth values \( a = T(p) \) and \( b = T(q) \) defines the truth value \( J(a, b) \) of the conditional proposition “if \( p \), then \( q \)”.

Fuzzy implications as extensions of the classical logic implication:

\[
\begin{align*}
\text{Crisp implication} & \quad a \implies b & \text{Fuzzy implication} & \quad J(a, b) \\
(S) & \quad \bar{a} \lor b & (S) & \quad u(c(a), b) \\
(R) & \quad \max \{ x \in [0, 1] \mid a \land x \leq b \} & (R) & \quad \sup \{ x \in [0, 1] \mid i(a, x) \leq b \} \\
(QL) & \quad \bar{a} \lor (a \land b) & (QL) & \quad u(c(a), i(a, b)) \\
(QL) & \quad (\bar{a} \land \bar{b}) \lor b & (QL) & \quad u(i(c(a), c(b)), b)
\end{align*}
\]
Fuzzy implications
Axiomatic requirements

Ax1. \( a \leq b \) implies \( J(a, x) \geq J(b, x) \) monoticity in first argument

Ax2. \( a \leq b \) implies \( J(x, a) \leq J(x, b) \) monotonicity in sec. arg.

Ax3. \( J(0, a) = 1 \) dominance of falsity

Ax4. \( J(1, b) = b \) neutrality of truth

Ax5. \( J(a, a) = 1 \) identity

Ax6. \( J(a, J(b, x)) = J(b, J(a, x)) \) exchange property

Ax7. \( J(a, b) = 1 \) iff \( a \leq b \) boundary condition

Ax8. \( J(a, b) = J(c(b), c(a)) \) contraposition

Ax9. \( J \) is a continuous function continuity

Fuzzy implications
How to select fuzzy implication

Criteria related to fuzzy inference rules
modus ponens, modus tollens, hypothetical syllogism.

Idea: If reduced to crisp sets, these rules should coincide with corresponding classical inference rules.

More formally: for fuzzy sets \( A(x), B(y) \) representing truth values by membership grades in \([0,1]\)

\[
B(y) = \sup_{x \in X} i(A(x), J(A(x), B(y))) \text{ modus ponens}
\]

\[
c(A(x)) = \sup_{y \in Y} i(c(B(y)), J(A(x), B(y))) \text{ modus tollens}
\]

\[
J(A(x), C(z)) = \sup_{y \in Y} i(J(A(x), B(y)), J(B(y), C(z))) \text{ hypoth. sylog.}
\]

should hold.

Look at Table 11.2, Table 11.3, and Table 11.4 (pp. 315-317).

One good choice:

\[
J_s(a, b) = \begin{cases} 
1 & a \leq b \\
0 & a > b 
\end{cases}
\]

One frequently used implication: Łukasiewicz

\[
J_a(a, b) = \min[1, 1 - a + b]
\]
Binary fuzzy relations

A super-brief introduction

- Binary fuzzy relations – definition Chapter 5.3

To represent (fuzzy) binary relations, membership matrices are convenient.

\[ R = [r_{xy}], \quad \text{where } r_{xy} = R(x, y). \]

An example:
Two fuzzy binary relations, \( P(X, Y) \) and \( Q(Y, Z) \) are given:

\[
\begin{align*}
P & = \begin{bmatrix}
0.3 & 0.5 & 0.8 \\
0.0 & 0.7 & 1.0 \\
0.4 & 0.6 & 0.5 
\end{bmatrix} \\
Q & = \begin{bmatrix}
0.9 & 0.5 & 0.7 & 0.7 \\
0.3 & 0.2 & 0.0 & 0.9 \\
1.0 & 0.0 & 0.5 & 0.5
\end{bmatrix}.
\end{align*}
\]

We read that, e.g.,
\[ \text{dom } P(x_2) = \max[0.0, 0.7, 1.0] = 1.0, \]
\[ \text{ran } Q(y_3) = \max[0.7, 0.0, 0.5] = 0.7. \]
**Binary fuzzy relations**

A super-brief introduction

We can also determine

\[ R = P \circ Q = [r_{ij}] = [p_{ik}] \circ [q_{kj}] = [\max_k \min(p_{ik}, q_{kj})] \]

\[
R =
\begin{bmatrix}
0.3 & 0.5 & 0.8 \\
0.0 & 0.7 & 1.0 \\
0.4 & 0.6 & 0.5
\end{bmatrix}
\circ
\begin{bmatrix}
0.9 & 0.5 & 0.7 & 0.7 \\
0.3 & 0.2 & 0.0 & 0.9 \\
1.0 & 0.0 & 0.5 & 0.5
\end{bmatrix}
\]

\[
= 
\begin{bmatrix}
0.8 & 0.3 & 0.5 & 0.5 \\
1.0 & 0.2 & 0.5 & 0.7 \\
0.5 & 0.4 & 0.5 & 0.6
\end{bmatrix}.
\]

For example

\[ t_{23} = \max[\min(0.0, 0.7), \min(0.7, 0.0), \min(1.0, 0.5)] \]
\[ = \max[0.0, 0.0, 0.5] = 0.5. \]

**Inference rules**

Fuzzy inference rules are basis for approximate reasoning.

As an example, three classical inference rules

(Modus ponens, Modus Tollens, Hypothetical syllogism)

are generalized by using **compositional rule of inference**

For a given fuzzy relation \( R \) on \( X \times Y \), and a given fuzzy set \( A' \) on \( X \), a fuzzy set \( B' \) on \( Y \) can be derived for all \( y \in Y \), so that

\[ B'(y) = \sup_{x \in X} \min[A'(x), R(x, y)]. \]

In matrix form, compositional rule of inference is

\[ B' = A' \circ R \]

**Approximate reasoning**

- Inference rules from conditional fuzzy propositions
  Chapter 8.6
- Multiconditional approximate reasoning
  Chapter 11.4

**Inference rules**

Fuzzy propositions as relations

The fuzzy relation \( R \) is, e.g., given by (one or more) conditional fuzzy propositions.

For a given fuzzy proposition

\[ p : \text{If } \mathcal{X} \text{ is } A, \text{ then } \mathcal{Y} \text{ is } B \]

a corresponding fuzzy relation is

\[ R(x, y) = \mathcal{J}[A(x), B(y)], \quad \text{for all } x \in X, y \in Y \]

where \( \mathcal{J} \) stands for a fuzzy implication.
We conclude that 

$\mathcal{Y} \text{ is } \mathcal{Y}'$.

In this case,

$$R(x, y) = \mathcal{J}[A(x), B(y)]$$

and

$$B'(y) = \sup \min_{x \in \mathcal{X}} [A'(x), R(x, y)].$$

**Inference rules**

**Generalized modus tollens**

**Rule:** If $\mathcal{X}$ is $A$, then $\mathcal{Y}$ is $B$

**Fact:** $\mathcal{Y}$ is $B'$

**Conclusion:** $\mathcal{X}$ is $A'$

In this case,

$$R(x, y) = \mathcal{J}[A(x), B(y)]$$

and

$$A'(x) = \sup \min_{y \in \mathcal{Y}} [B'(y), R(x, y)].$$

**Example:**

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be the sets of values of variables $\mathcal{X}, \mathcal{Y}$.

Let $A = 0.5/x_1 + 1/x_2 + 0.6/x_3$ and $B = 1/y_1 + 0.4/y_2$.

Let $A' = 0.6/x_1 + 0.9/x_2 + 0.7/x_3$.

Let $R(x, y) = \mathcal{J}[A(x), B(y)] = \min[1, 1 - A(x) + B(y)]$.

By using Generalized modus tollens, derive the conclusion $\mathcal{X}$ is $A'$.

We compute:

$$R = 1/x_1 + 1/x_2 + 1/x_3 + 0.4/x_2 + 0.4/x_2 + 1/x_3, y_1 + 0.8/x_3, y_2$$

$$A'(x_1) = \sup \min_{y \in \mathcal{Y}} [B'(y), R(x, y)]$$

$$= \max[\min(0.9, 1), \min(0.7, 0.9)] = \max[0.9, 0.7] = 0.9$$

$$A'(x_2) = \sup \min_{y \in \mathcal{Y}} [B'(y), R(x, y)]$$

$$= \max[\min(0.9, 1), \min(0.7, 0.4)] = \max[0.9, 0.4] = 0.9$$

$$A'(x_3) = \sup \min_{y \in \mathcal{Y}} [B'(y), R(x, y)]$$

$$= \max[\min(0.9, 1), \min(0.7, 0.8)] = \max[0.9, 0.7] = 0.9$$

We conclude that $A' = 0.9/x_1 + 0.9/x_2 + 0.9/x_3$. 

Approximate Reasoning

Introduction

Fuzzy Sets and Fuzzy Techniques
Inference rules

Generalized hypothetical syllogism

For variables \( \mathcal{X}, \mathcal{Y}, \mathcal{Z} \) taking values from sets \( X, Y, Z \) respectively, and \( A, B, C \) being fuzzy sets on \( X, Y, Z \), respectively:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 1:</td>
<td>If ( \mathcal{X} ) is ( A ), then ( \mathcal{Y} ) is ( B )</td>
</tr>
<tr>
<td>Rule 2:</td>
<td>If ( \mathcal{Y} ) is ( B ), then ( \mathcal{Z} ) is ( C )</td>
</tr>
</tbody>
</table>

**Conclusion:** If \( \mathcal{X} \) is \( A \), then \( \mathcal{Z} \) is \( C \)

In this case, three relations are defined:

\[
R_1(x, y) = \mathcal{J}[A(x), B(y)] \\
R_2(y, z) = \mathcal{J}[B(y), C(z)] \\
R_3(x, z) = \mathcal{J}[A(x), C(z)].
\]

The generalized hypothetical syllogism holds if

\[
R_3(x, z) = \sup_{y \in Y} \min[R_1(x, y), R_2(x, y)]
\]

or, in matrix notation, if

\[
R_3 = R_1 \circ R_2.
\]

Multifocal approximate reasoning

**General schema** is of the form:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule 1:</td>
<td>If ( \mathcal{X} ) is ( A_1 ), then ( \mathcal{Y} ) is ( B_1 )</td>
</tr>
<tr>
<td>Rule 2:</td>
<td>If ( \mathcal{X} ) is ( A_2 ), then ( \mathcal{Y} ) is ( B_2 )</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>Rule n:</td>
<td>If ( \mathcal{X} ) is ( A_n ), then ( \mathcal{Y} ) is ( B_n )</td>
</tr>
</tbody>
</table>

**Fact:** \( \mathcal{X} \) is \( A' \)

If \( A', A_j \) are fuzzy sets on \( X \), \( B', B_j \) are fuzzy sets on \( Y \), for all \( j \).

Example:

Let \( X = \{x_1, x_2, x_3\} \), \( Y = \{y_1, y_2\} \), and \( Z = \{z_1, z_2\} \) be the sets of values of variables \( \mathcal{X}, \mathcal{Y}, \mathcal{Z} \).

Let \( A = 0.5/x_1 + 1/x_2 + 0.6/x_3 \), \( B = 1/y_1 + 0.4/y_2 \), \( C = 0.2/z_1 + 1/z_2 \).

Let

\[
R(x, y) = \mathcal{J}[A(x), B(y)] = \begin{cases} 
1 & a \leq b \\
0 & a > b \end{cases}
\]

Check if generalized hypothetical syllogism holds.

We write

\[
R_1 = \begin{bmatrix} 1 & 0.4 \\ 1 & 0.4 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.2 & 1 \\ 0.2 & 1 \end{bmatrix}, \quad R_3 = \begin{bmatrix} 0.2 & 1 \\ 0.2 & 1 \end{bmatrix}
\]

and we check that \( R_1 \circ R_2 = R_3 \).

Multifocal approximate reasoning

**Method of interpolation**

Most common way to determine \( \mathcal{B}' \) is by using method of interpolation.

**Step 1.** Calculate the degree of consistency between the given fact and the antecedent of each rule.

Use height of intersection of the associated sets:

\[
r'_j(A') = h(A' \land A_j) = \sup_{x \in X} \min[A'(x), A_j(x)].
\]

**Step 2.** Truncate each \( B_j \) by the value \( r'_j(A') \) and determine \( \mathcal{B}' \) as the union of truncated sets:

\[
\mathcal{B}'(y) = \sup_{j \in \mathbb{N}_n} \min[r'_j(A'), B_j(y)], \quad \text{for all } y \in Y.
\]

Note that interpolation method is a special case of the composition rule of inference, with

\[
R(x, y) = \sup_{j \in \mathbb{N}_n} \min[A_j(x), B_j(y)]
\]

where then \( \mathcal{B}'(y) = \sup_{x \in X} \min[A'(x), R(x, y)] = (A' \circ R)(y) \).
Multiconditional approximate reasoning

Method of interpolation-Example

An application
Region growing based on fuzzy rule based system


Fig. 7: An example for the evaluation of the fuzzy rule-based homogeneity criterion.