Community Ordered Formation Theory and its Applications in Image Analysis

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www.cb.uu.se/~filip/ImageProcessingUsingGraphs/schedule.html

Objective

This talk presents a methodology, which has been very well succeeded in Image Analysis, from a more general point of view, in order to invite collaborators from other research areas.

- The Community Ordered Formation process, where groups of individuals are formed based on optimum connectivity relations to their leaders.

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- The mathematical and computational models of the COF process.

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- Conclusive remarks.

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- The individuals with higher desire offer to their acquaintances a reward to be part of their community.
- If the offered reward is higher than his/her current reward/desire, then the acquaintance agrees to change community.

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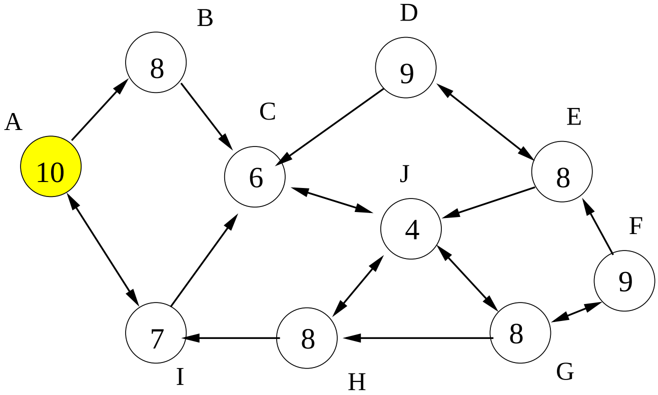
- The rewards are propagated from the true leaders through the members of their communities, which always offer a reward not higher than their own reward.
- The population is divided into communities, where each individual belongs to the group which offered to him/her the highest reward.

Community Ordered Formation Theory



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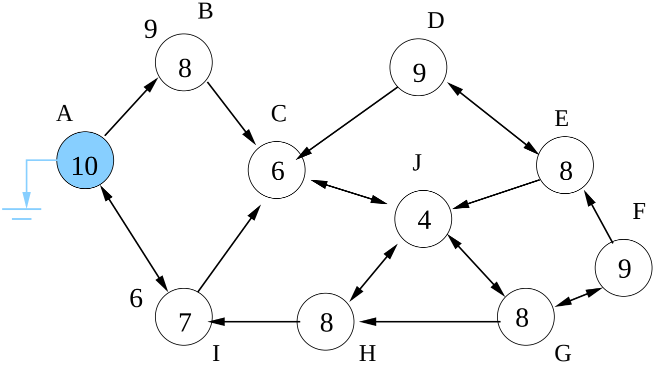

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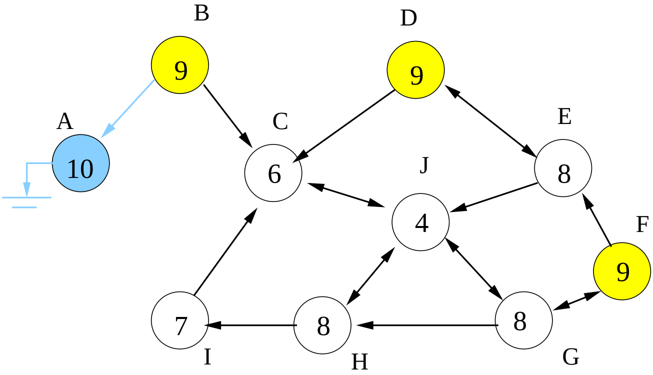

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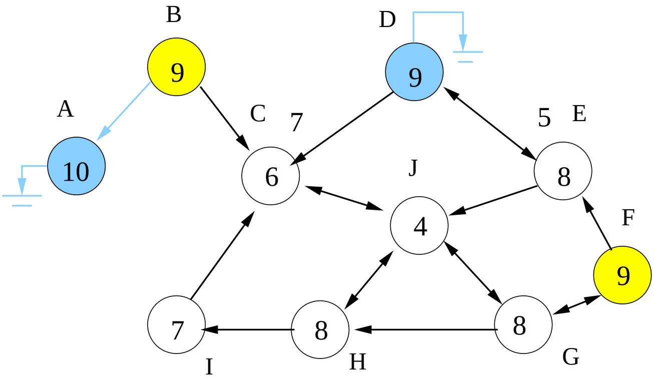

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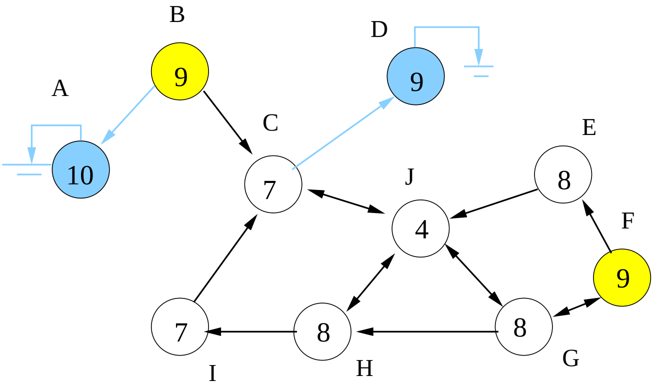

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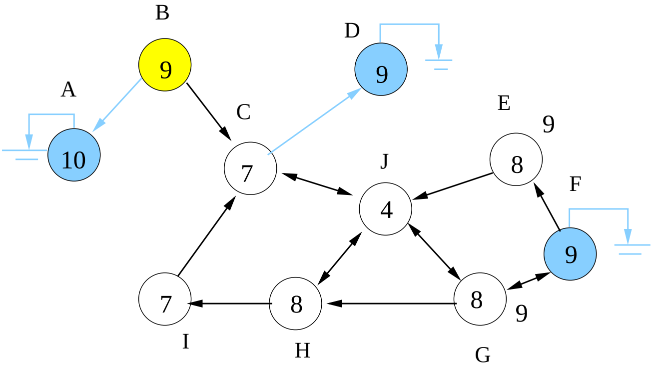

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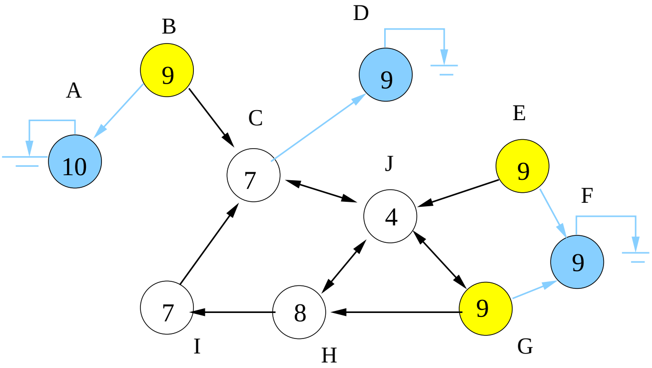

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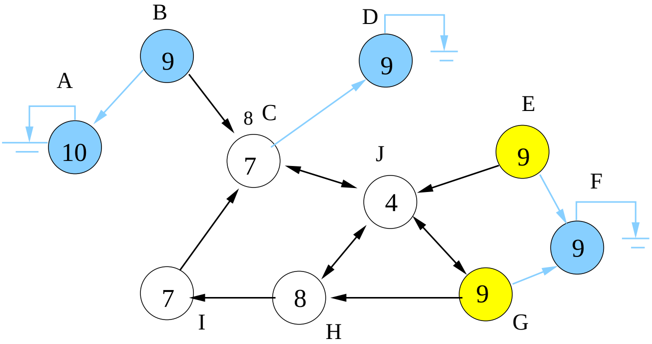

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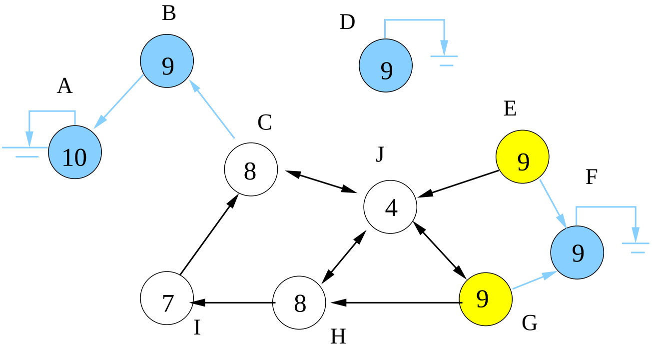

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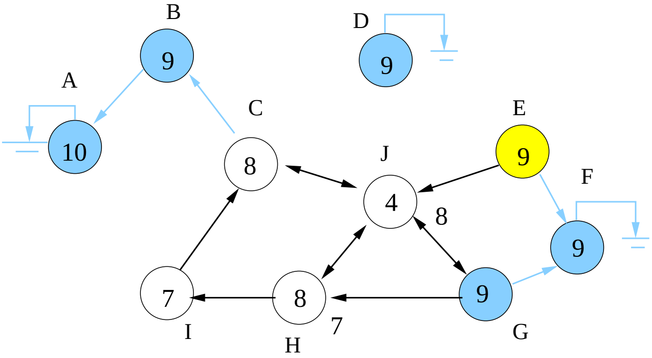

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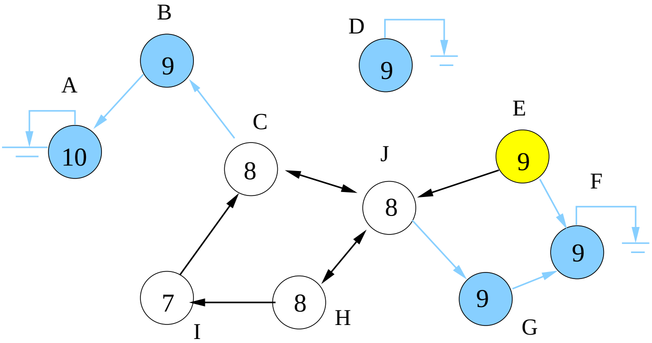

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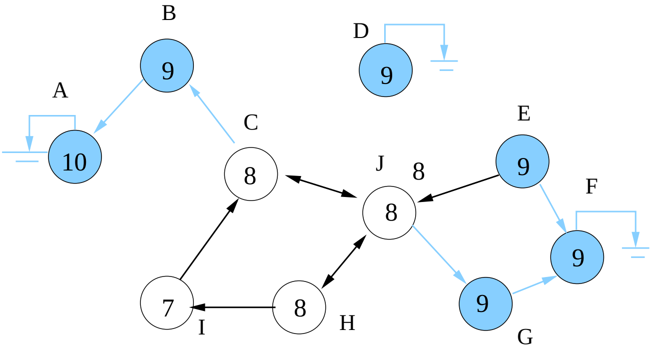

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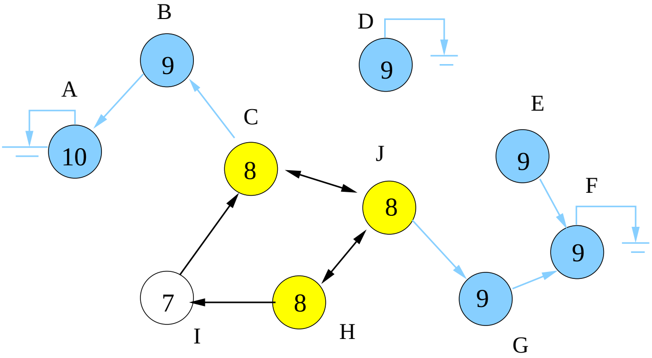

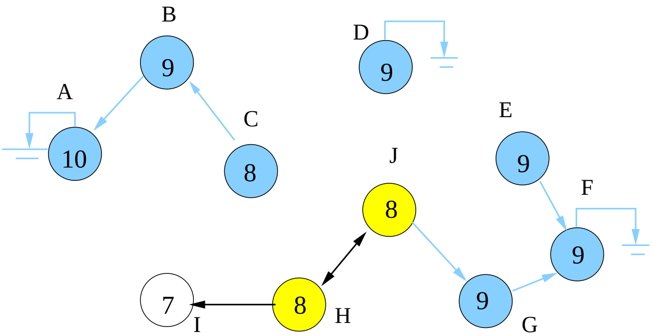
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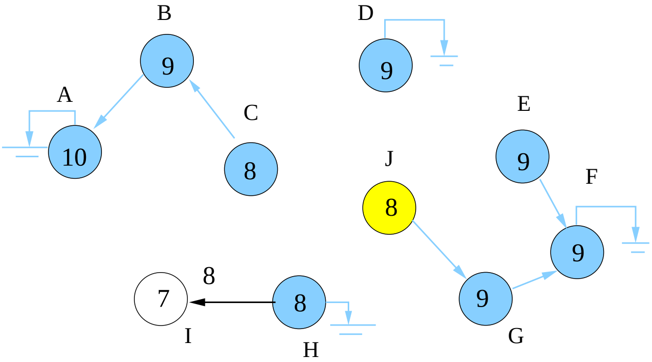

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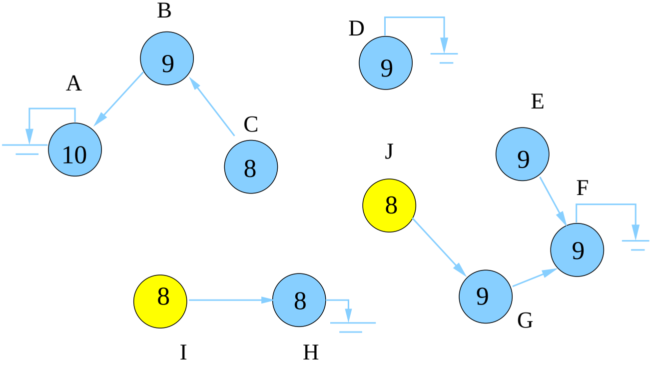

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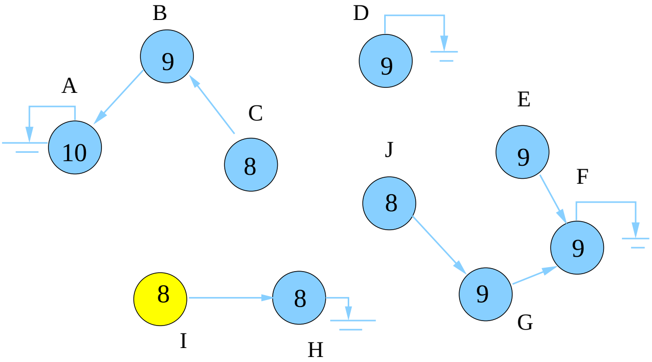

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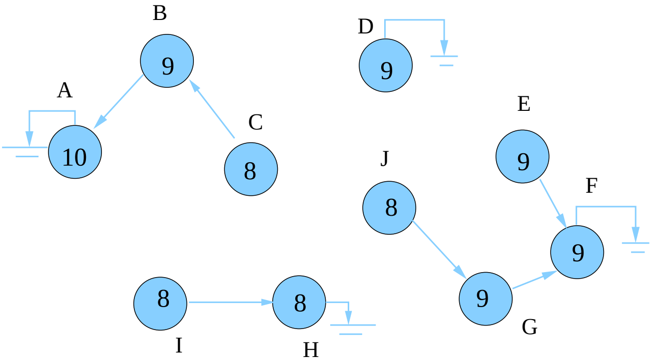

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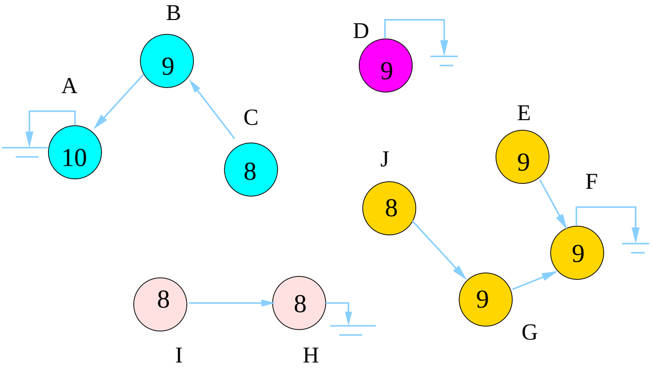

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- A sequence of invitations, starting at a leader s_1, passing through other individuals, and ending at an individual $s_n = t$, forms a simple path $\pi_t = \langle s_1, s_2, \ldots, s_n \rangle$, where $(s_i, s_{i+1}) \in \mathcal{A}$.

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- Solitary individuals $\pi_t = \langle t \rangle$ form trivial paths.

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$$V(t) = \max_{\forall \pi_t \in \Pi(\mathcal{N}, \mathcal{A}, t)} \{f(\pi_t)\},$$

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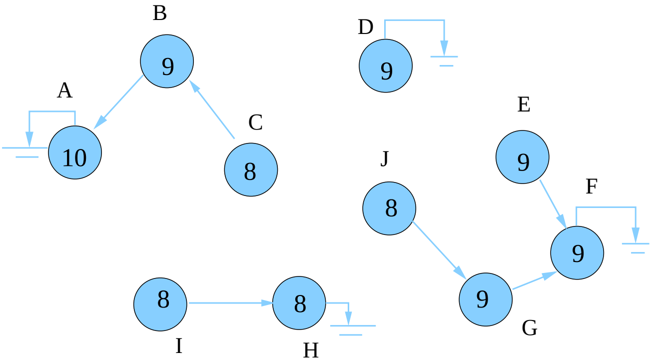
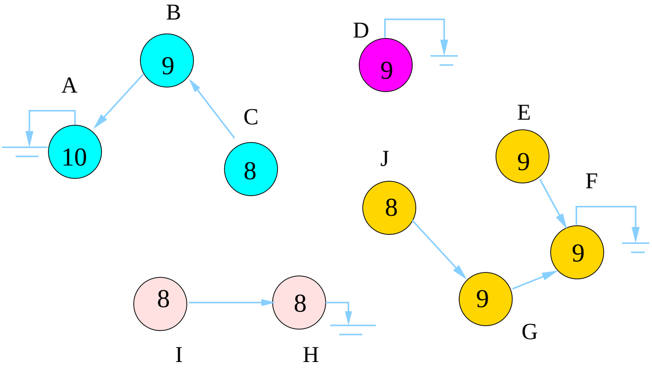
However, this process follows the non-increasing order of optimum connectivity (reward) values.

A generalization of Dijkstra's algorithm solves this problem by outputting an optimum-path forest P — i.e., an acyclic map that assigns a mark $nil \notin \mathcal{N}$ to every individual $t \in \mathcal{N}$, when t is a leader (root of the forest), or a predecessor $P(t) = s \in \mathcal{N}$ in the optimum path $P^*(t)$.



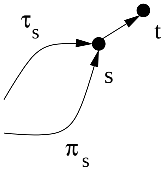
As subproducts, the COF algorithm also outputs the maximum connectivity map $V(t)$ and an optimum partition $R(t)$, which assigns to each individual t its root (leader) $R(t)$ or any other label $L(t)$ associated with $R(t)$.



Correctness

The correctedness of the COF algorithm requires that for every $t \in \mathcal{N}$, there must exist at least one optimum path π_t, either trivial or simple $\pi_t = \pi_s \cdot \langle s, t \rangle$, such that:

1. $f(\pi_s) \geq f(\pi_t)$.
2. The prefix π_s is optimum.
3. For any other optimum prefix τ_s, $f(\tau_s \cdot \langle s, t \rangle) = f(\pi_t)$.



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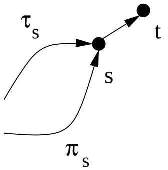

These conditions are only applied to optimum paths.

Applications in Image Analysis

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- For pixels, the COF process is called an Image Foresting Transform (IFT), whose seminal work was published in [17].
- A COF-based image operator requires an adjacency relation, which may be defined in the image domain and/or in the feature space, and a connectivity function.

- Maximizing (minimizing) $V(t)$ with the minimum (maximum) arc weight along the paths.

$$
\begin{aligned}
f_{\min}(\langle t \rangle) &= H(t) \\
f_{\min}(\pi_s \cdot \langle s, t \rangle) &= \min\{f_{\min}(\pi_s), w(s,t)\}
\end{aligned}
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Connectivity Functions

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- Minimizing $V(t)$ with the sum of arc weights along the paths.

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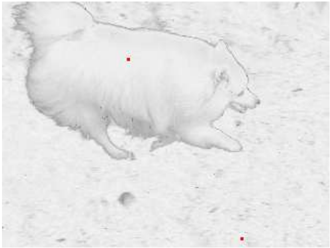
- Minimizing $V(t)$ with the Euclidean distance between the terminal nodes of the paths.

$$\begin{aligned} f_{euc}(\langle t \rangle) &= \begin{cases} 0 & \text{if } t \in \mathcal{S} \\ +\infty & \text{otherwise} \end{cases} \\ f_{euc}(\pi_s \cdot \langle s, t \rangle) &= \|t - R(s)\| \end{aligned}$$

Pixel Clustering

Random samples can be used to estimate a probability density function (pdf) with a few maxima (true leaders) and one optimum-path tree rooted at each maximum defines a cluster.

Pixel Clustering



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Background Removal

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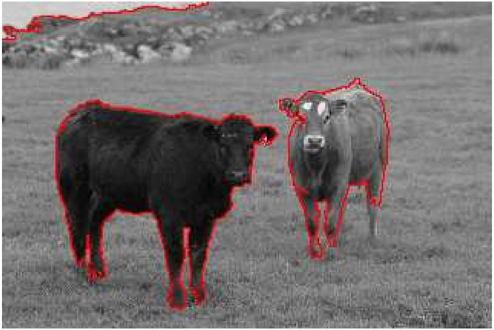
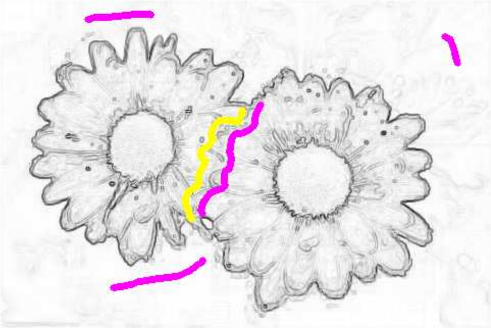


Random samples from the image's border can be used to estimate the pdf of the background, reducing segmentation to an optimum thresholding on the density values.

Optimum connectivity with markers

Object and background markers compete for the most strongly connected pixels. The strength of connectedness is reduced when paths cross the object's borders. The ordering process guarantees connected regions.



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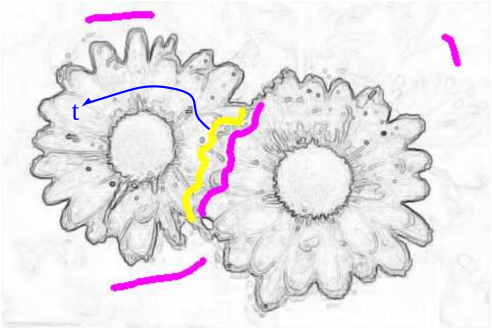

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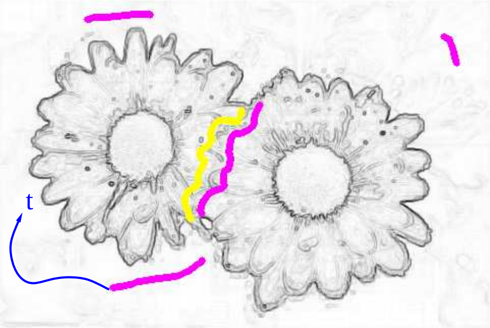

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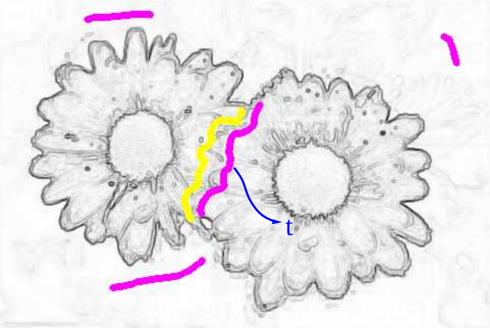

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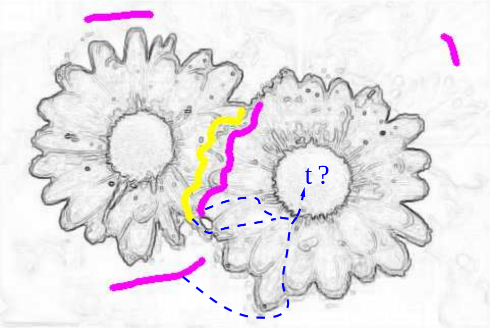

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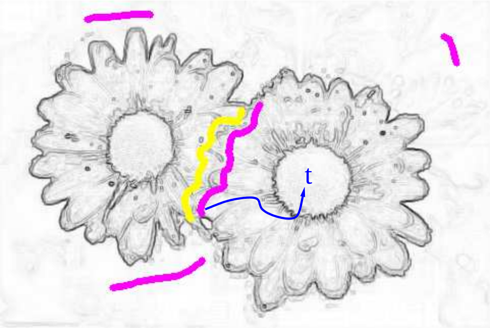

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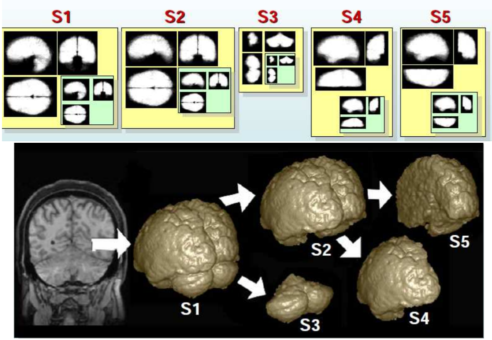
Combination with object models

Medical imaging: Object modeling and image segmentation



Object models can be used to estimate internal and external markers for automatic segmentation. Clustering completes segmentation inside the objects.

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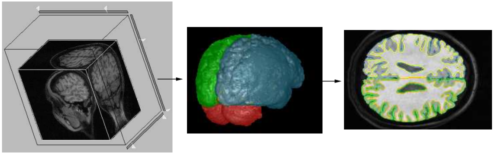

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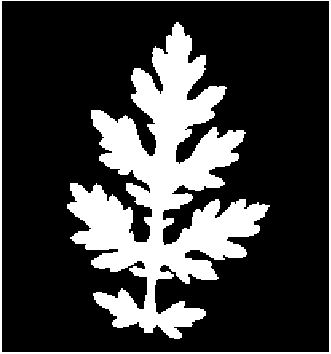
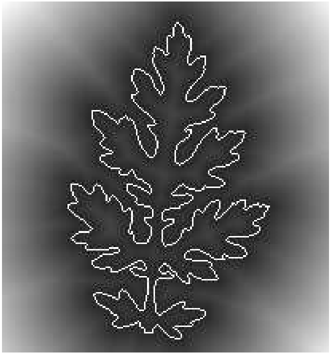


- Shapes can be represented in multiple scales.

Multiscale Shape Representation



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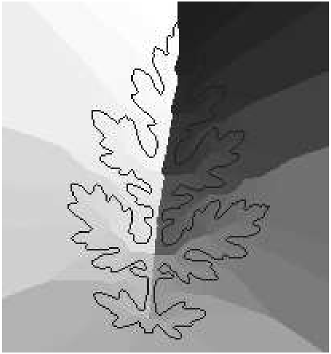

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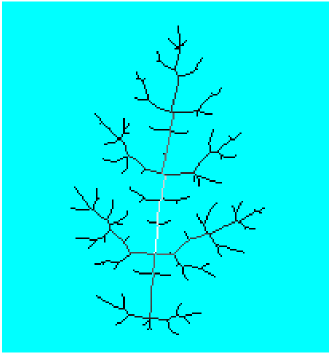

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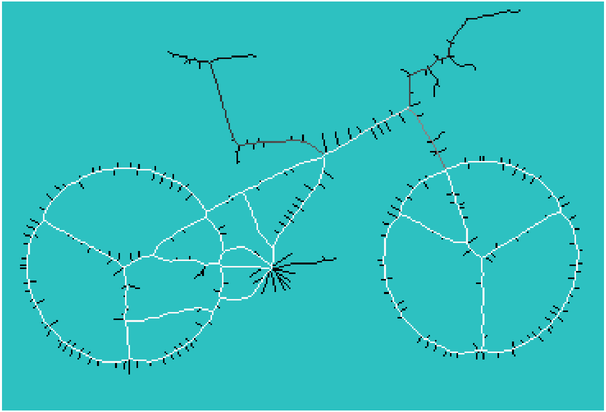
- The root map creates discrete Voronoi regions.
- Multiscale skeletons are obtained from the roop map, by computing geodesic distances along the contour between the roots of 4-adjacent pixels.

Multi-Scale Skeletons

The skeletons are one-pixel wide and connected in all scales and a proper scale can be chosen before it disconnects from the SKIZ.



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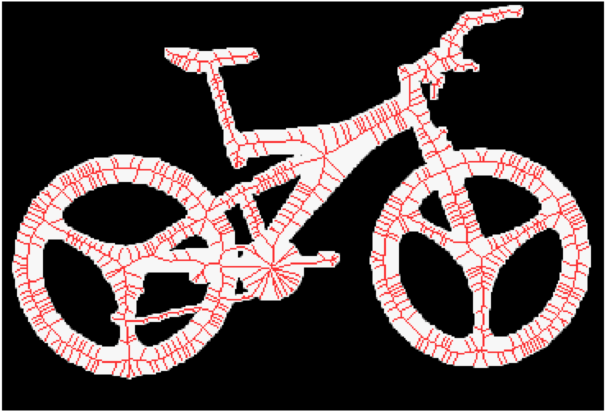

The 3D extension exploits geodesic areas[18].

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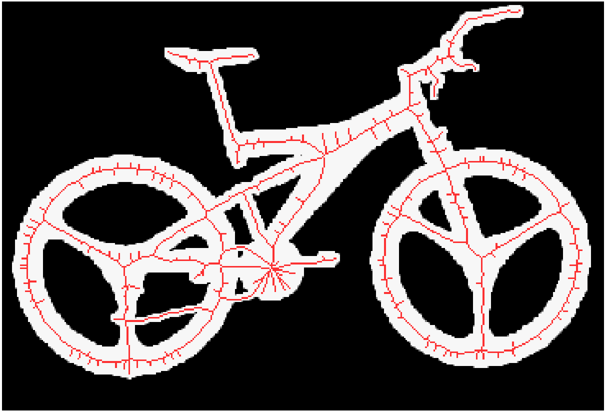

The 3D extension exploits geodesic areas[18].

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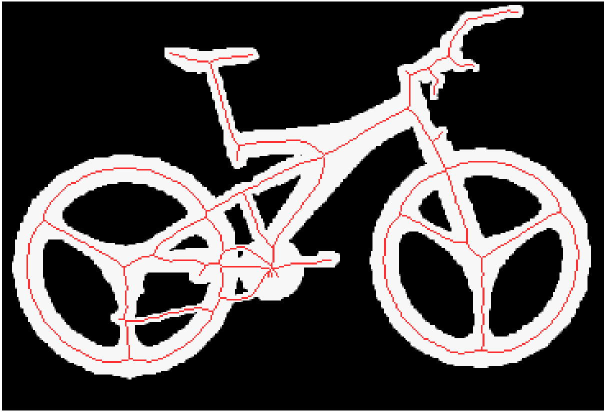

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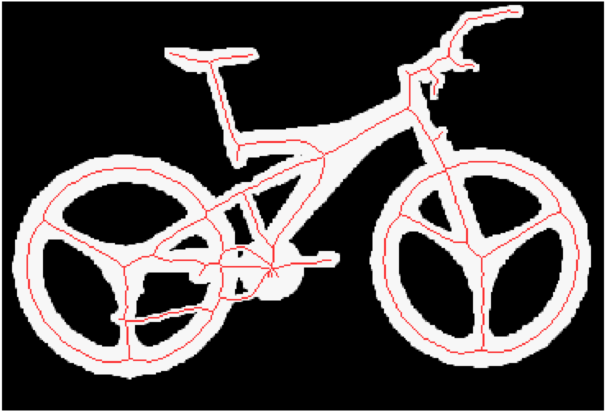

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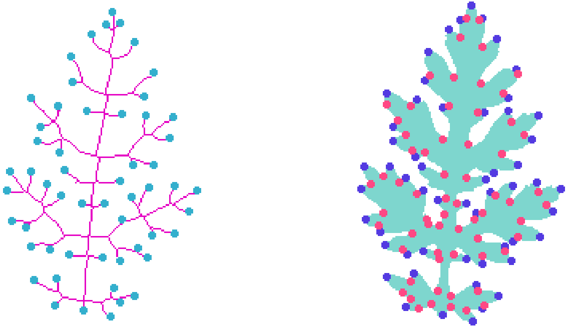
Contour Saliences



The internal and external skeleton saliences lead to the convex and concave contour saliences, respectively.

Image Classification

An image classifier can be created by active learning to reduce the number of relevant feedback iterations for Contend-Based Image Retrieval.

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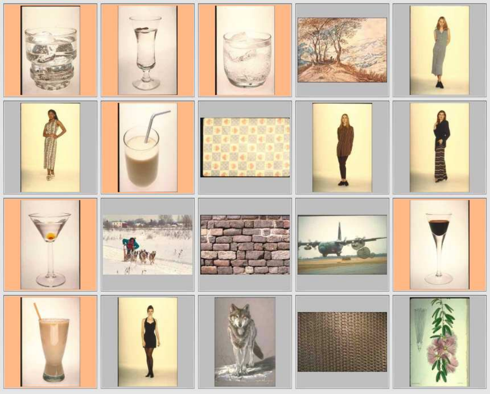

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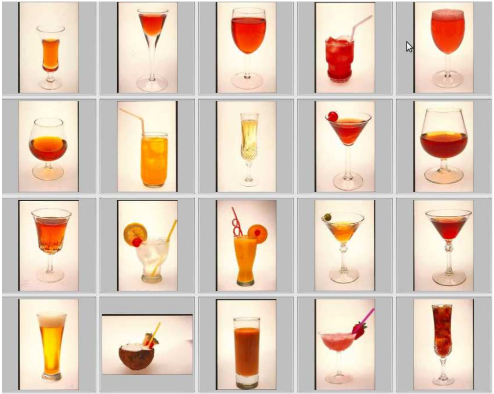

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Conclusion

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- All image operators have been implemented with a few types of connectivity functions. Can we increase this small set of functions?
- Can we include dynamics to the COF process, by analyzing changes along time on the optimum-path forest?
- Can we allow an individual to be part of multiple communities and use this methodology in new applications?

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