# Image Foresting Transform

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- connectivity relations: sequences of adjacent pixels (e.g., component labeling).

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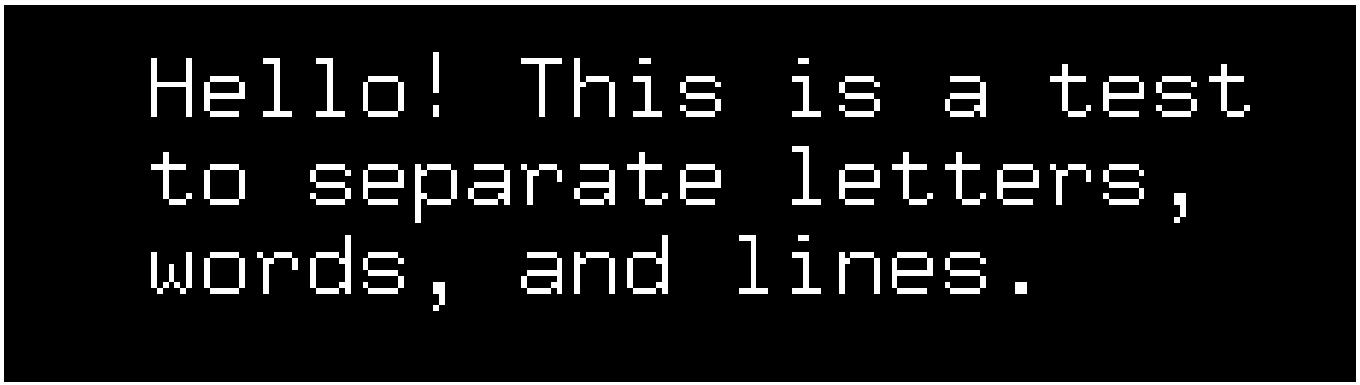
- The graph nodes may be pixels, edges, regions, and the arcs will result from a given adjacency relation.
- This strategy counts with several algorithms from Graph Theory and their proof of correctness.

The same algorithm with distinct adjacency relations, for example, can label letters, words and lines.



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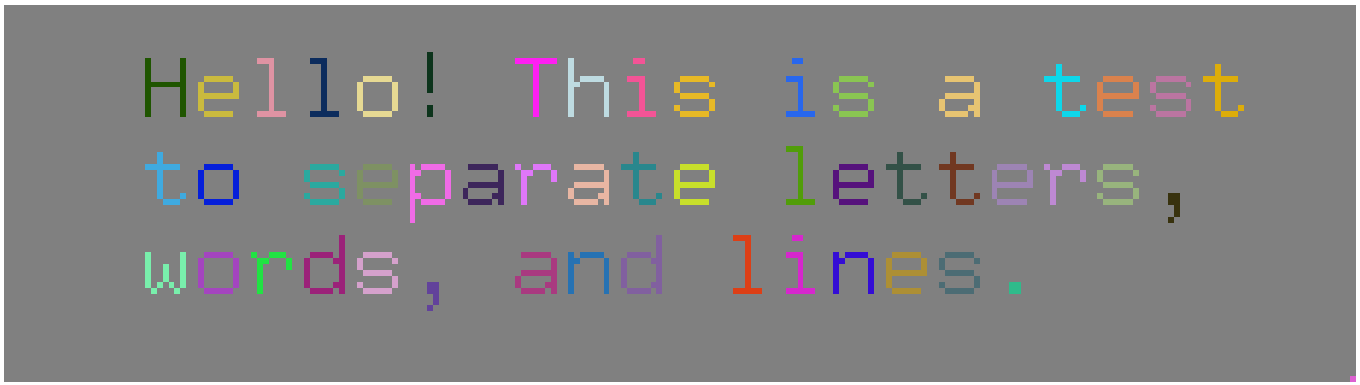

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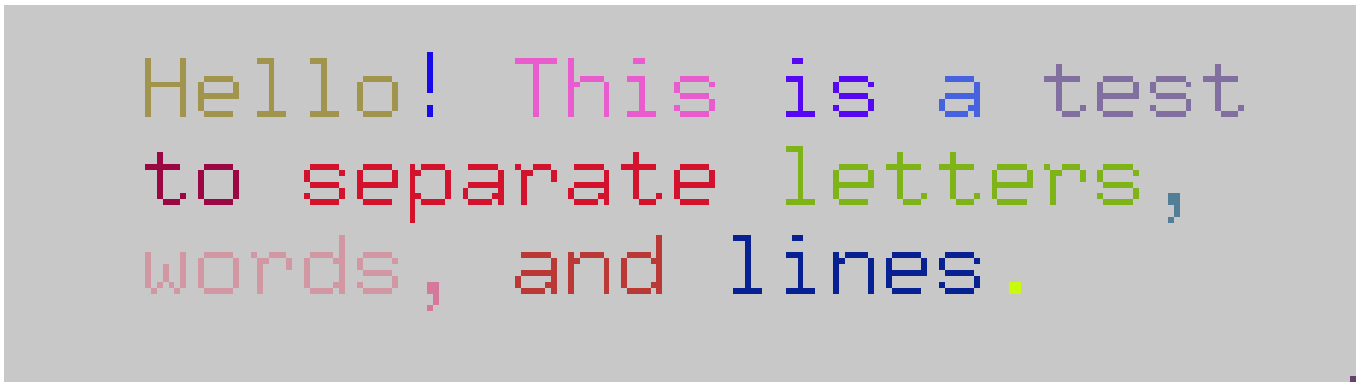

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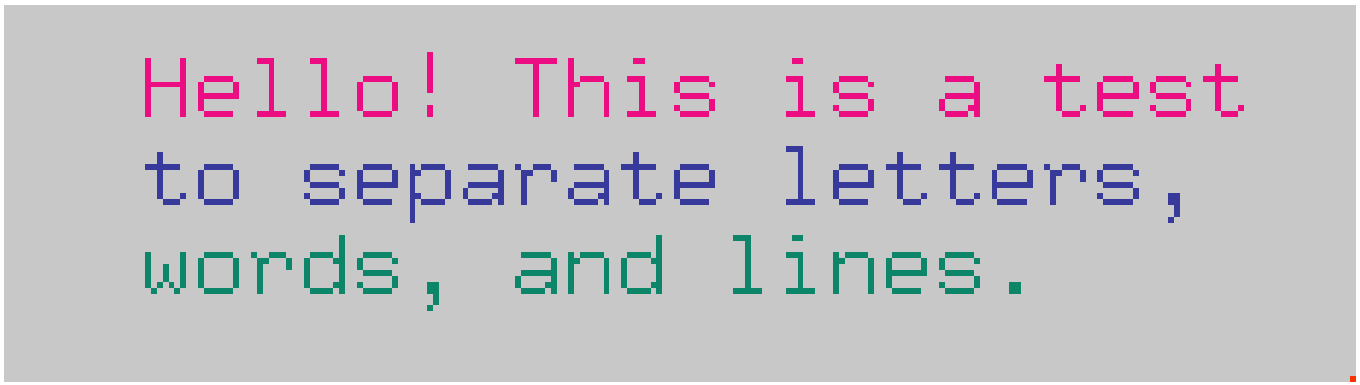

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- linear-time algorithms for most applications [2, 3],
- unified framework to improve methods [4, 5, 6, 7, 8], including hardware-based implementations [9, 10], and
- effective solutions for segmentation [11, 12, 13, 14, 15], filtering [4], representation [16], description [17, 18, 19], clustering [20], and classification [21, 22, 23].

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- Basic definitions.

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- General algorithm, variants, and implementation issues.

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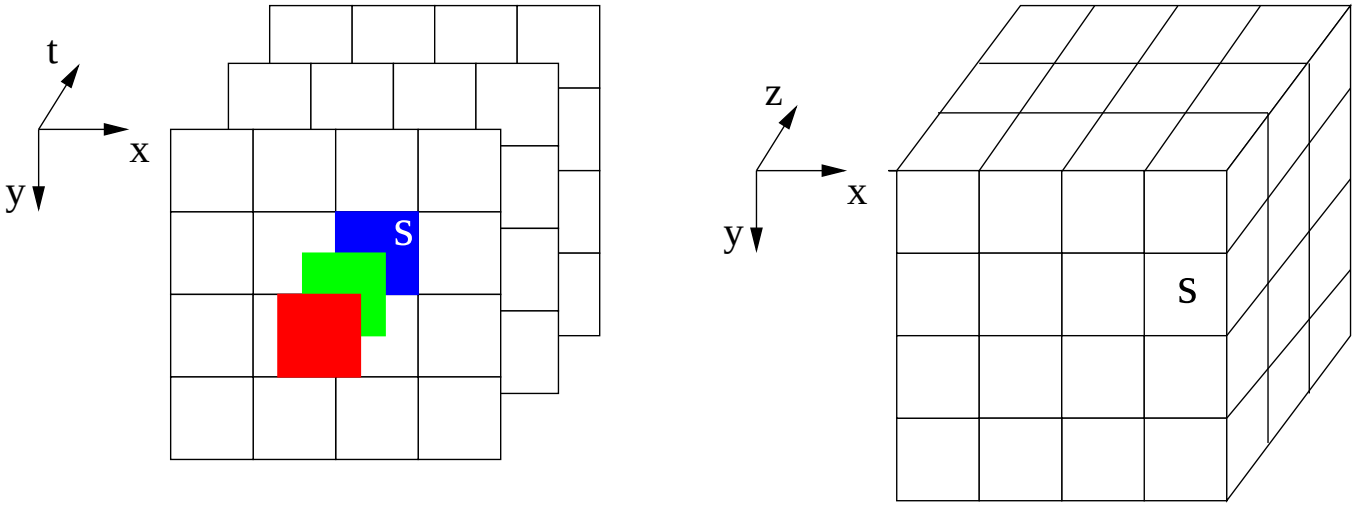
- $\vec{I}(s) = (I_1(s), I_2(s), \ldots, I_m(s)) \in \mathcal{Z}^m$ is a vectorial mapping, which assigns a set of values to each $s \in \mathcal{D}_I$.

For $m = 1$, we use $\mathbf{I} = (\mathcal{D}_I, I)$.

# General image definition

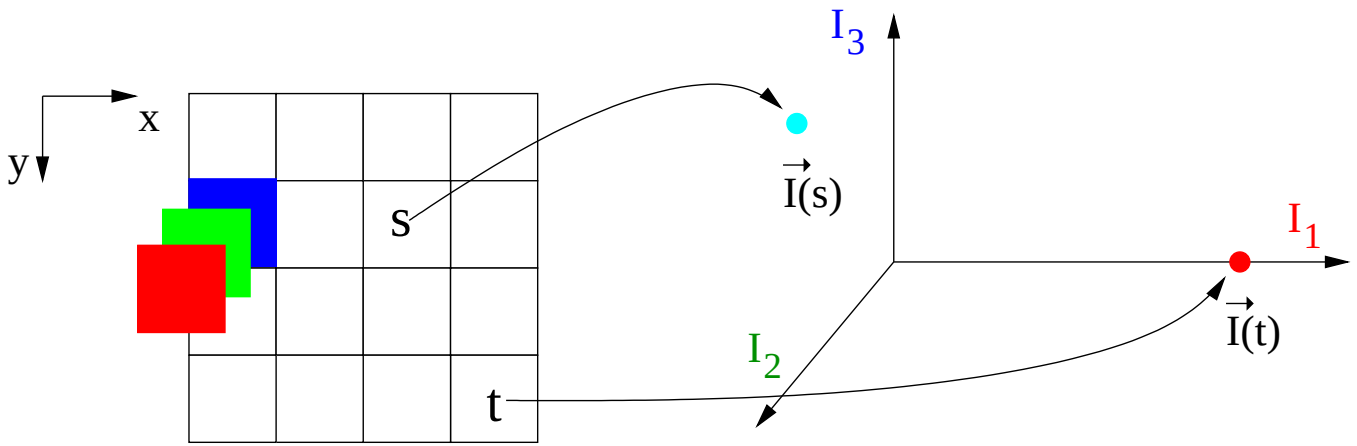



(a) (b)

(a) A RGB video $\mathbf{I}$, $n = 3$ and $m = 3$. (b) A CT image $\mathbf{I}$, $n = 3$ and $m = 1$.

# Image domain and feature space

A spel $s \in \mathcal{D}_I$ is a point $\vec{I}(s) \in \mathcal{Z}^m$ in the feature space.



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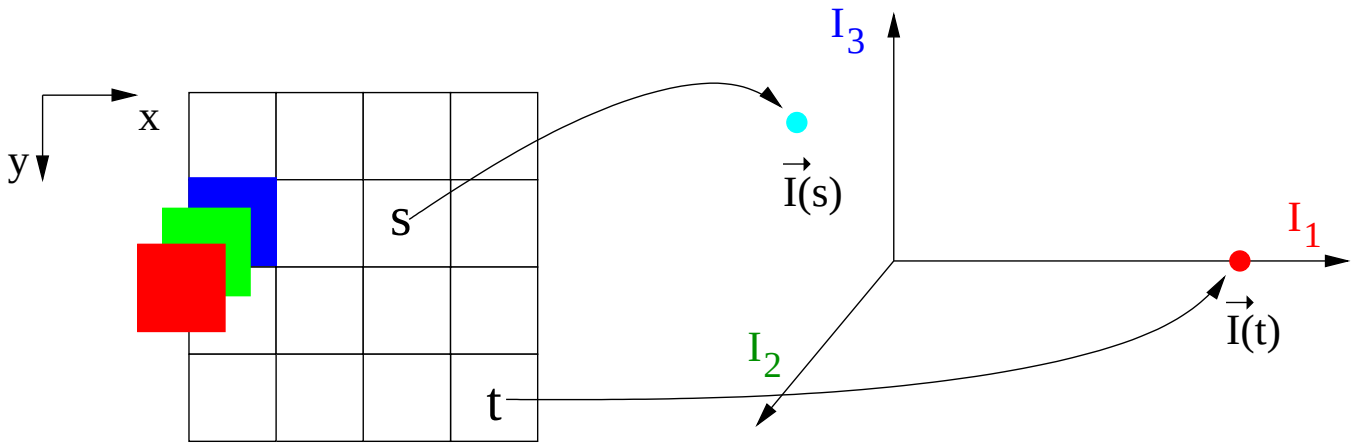


New features may result from any image transformation $\Psi(\mathbf{I})$, creating a real image $\mathbf{F} = (\mathcal{D}_F, \vec{F})$ where $\mathcal{D}_F = \mathcal{D}_I$ and $\vec{F}(s) = (F_1, F_2, \ldots, F_{m'}) \in \Re^{m'}$.

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We will indicate that a spel $t$ is adjacent to spel $s$ either by $(s, t) \in \mathcal{A}$ or $t \in \mathcal{A}(s)$.

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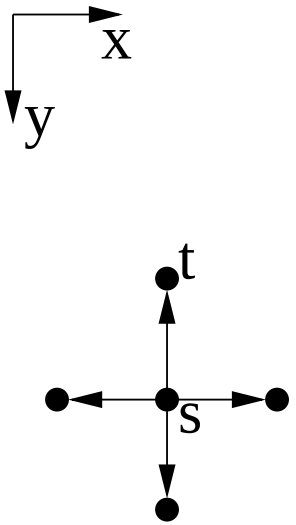
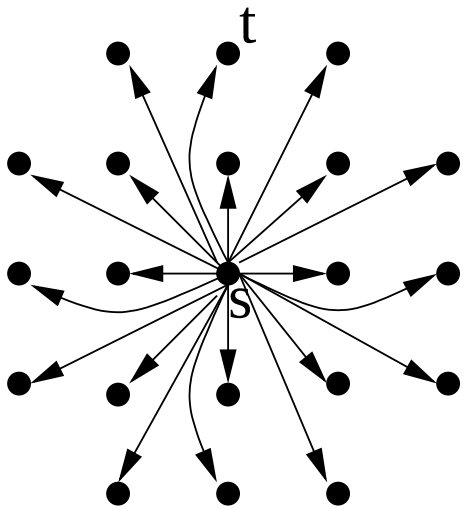
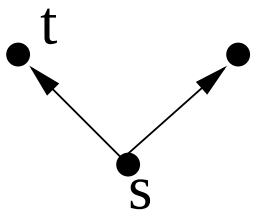



(a) Ex 1: r = 1




(b) Ex 1: r = $\sqrt{5}$

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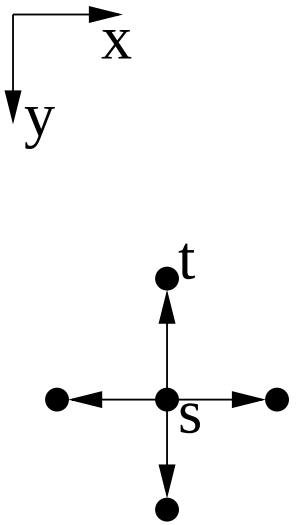


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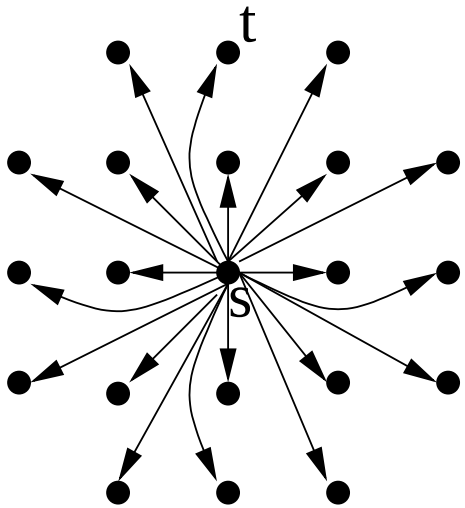


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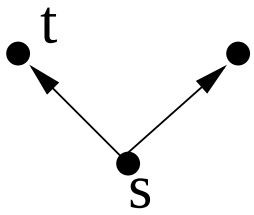


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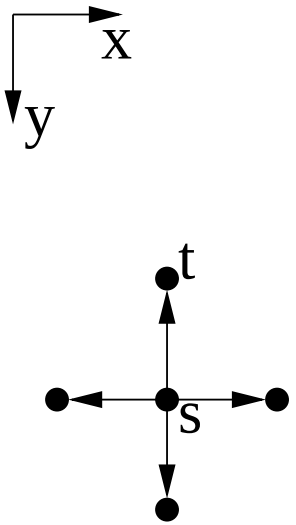


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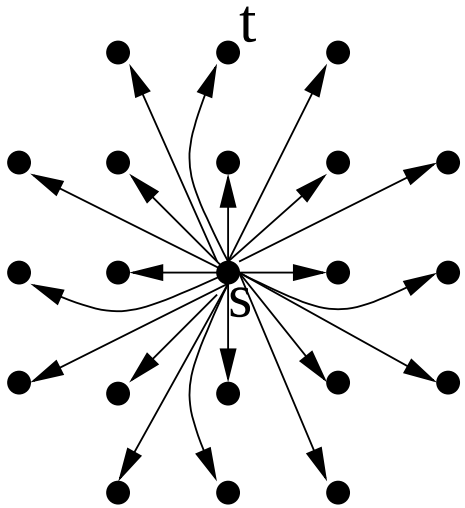


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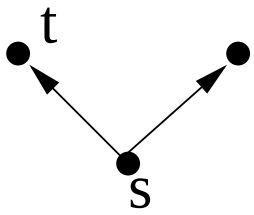


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- Example 3: $(s,t) \in \mathcal{A}$ when $t$ is a $k$-nearest neighbor of $s$ in the feature space, for $k \geq 1$.

# Images as graphs

Position invariant relations $\mathcal{A}$ can be represented by a vector of relative displacements

$$t - s \quad \in \quad \{(dx_1, dy_1), (dx_2, dy_2), \ldots, (dx_d, dy_d)\},$$

and fixed size $d = |\mathcal{A}(s)|$ $\forall s$, leading to an implicit graph representation.

$$(x_t, y_t) \quad = \quad (x_s, y_s) + (dx_i, dy_i), i = 1, 2, \ldots, d,$$

where $t = (x_t, y_t)$ and $s = (x_s, y_s)$.

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- The dual definition $f(\pi_t) \geq f(\tau_t)$ (maximum) is also valid.

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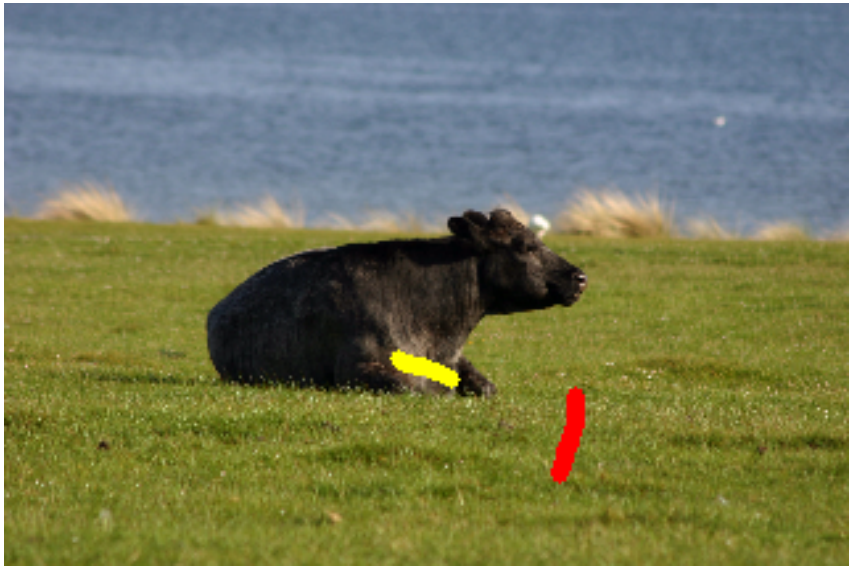

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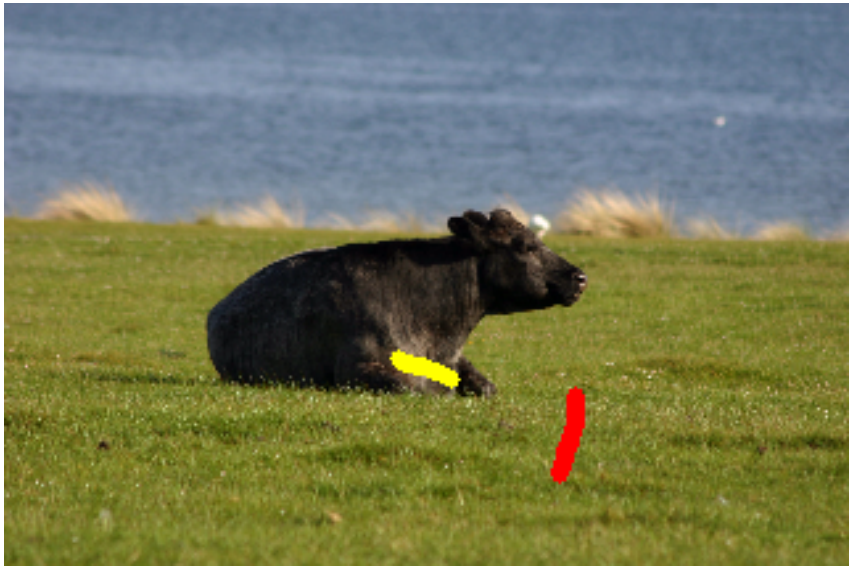

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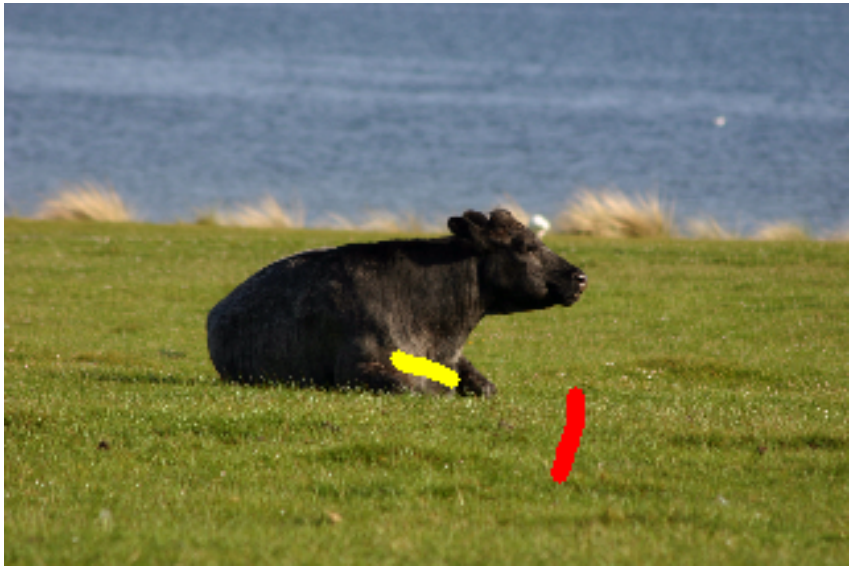

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- The seeds may compete among themselves by offering optimum paths to every spel in the image.
- The object can be defined by spels $t$ whose optimum paths $\pi_t$ have roots $R(\pi_t)$ in $\mathcal{S}_i$.

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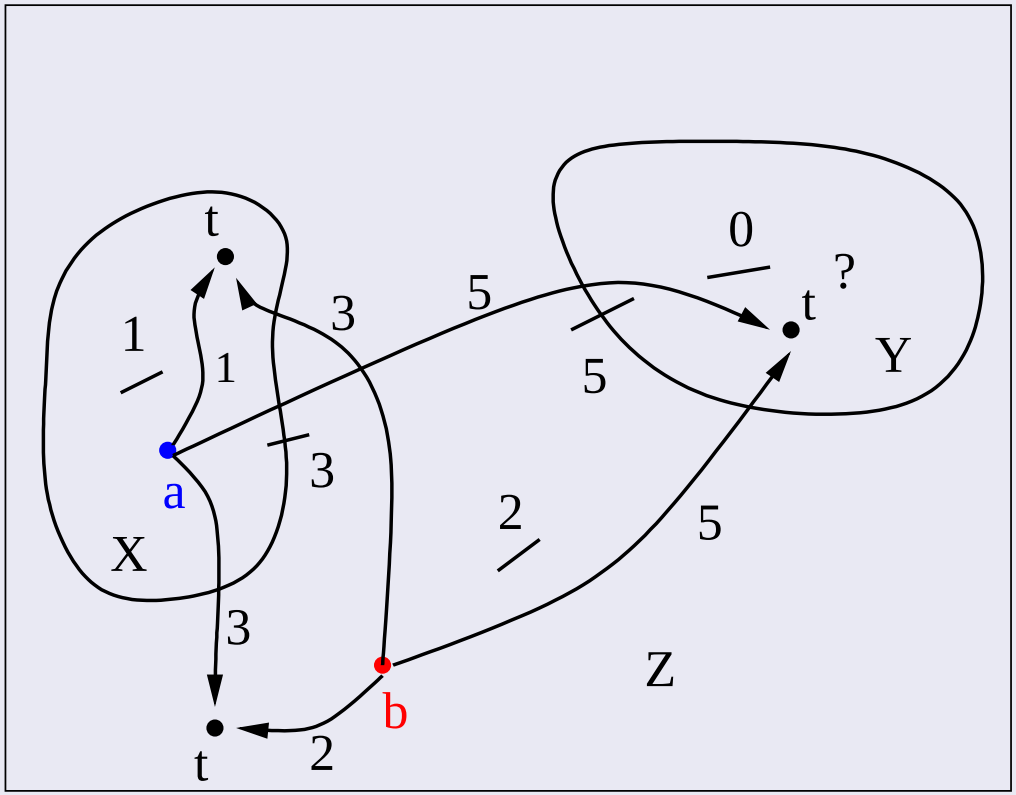
- define the value of a path to be the maximum arc weight along it, such that any path that crosses the boundary will be penalized.

# The IFT computation

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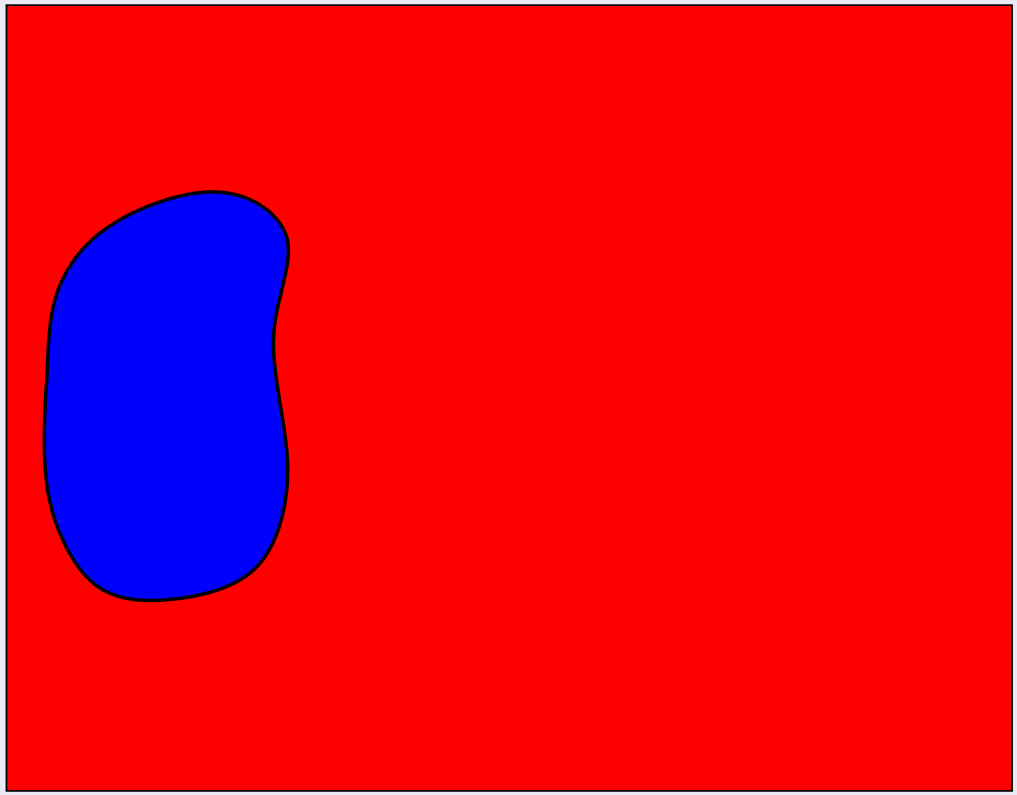

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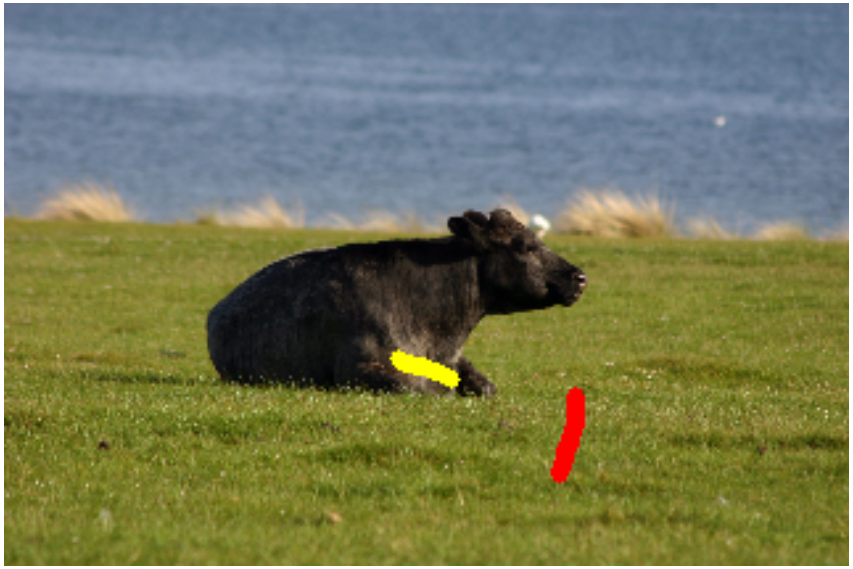

- Image with internal and external markers.

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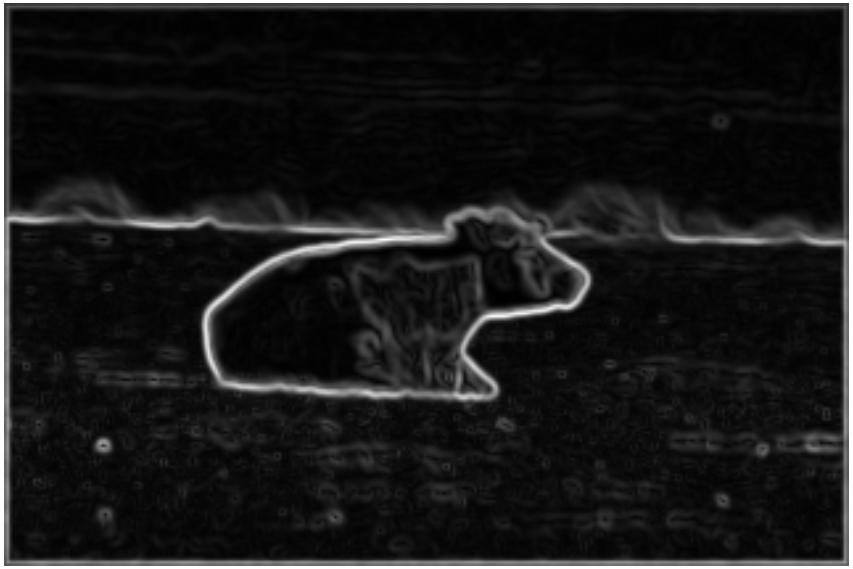
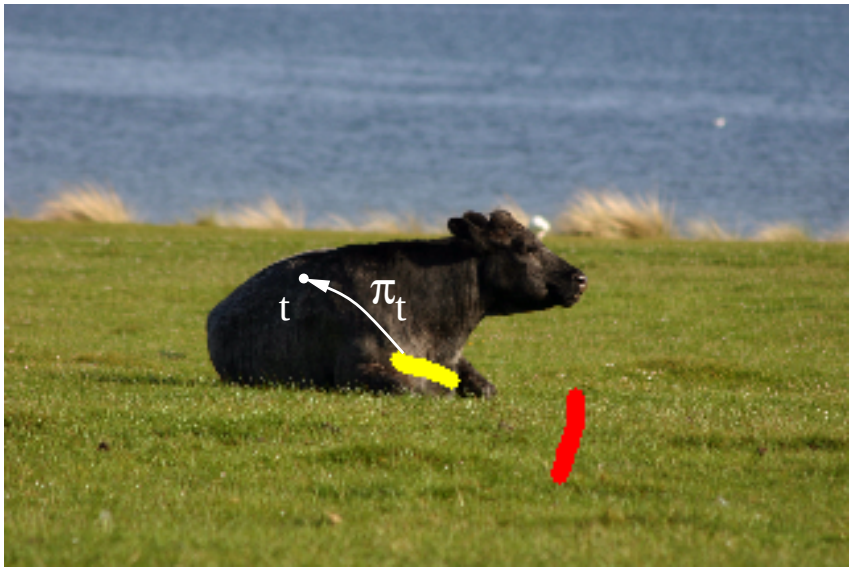


- Image with internal and external markers.
- Arc-weight image.

# Connectivity functions




- Image with internal and external markers.
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- Optimum-paths to foreground pixels.

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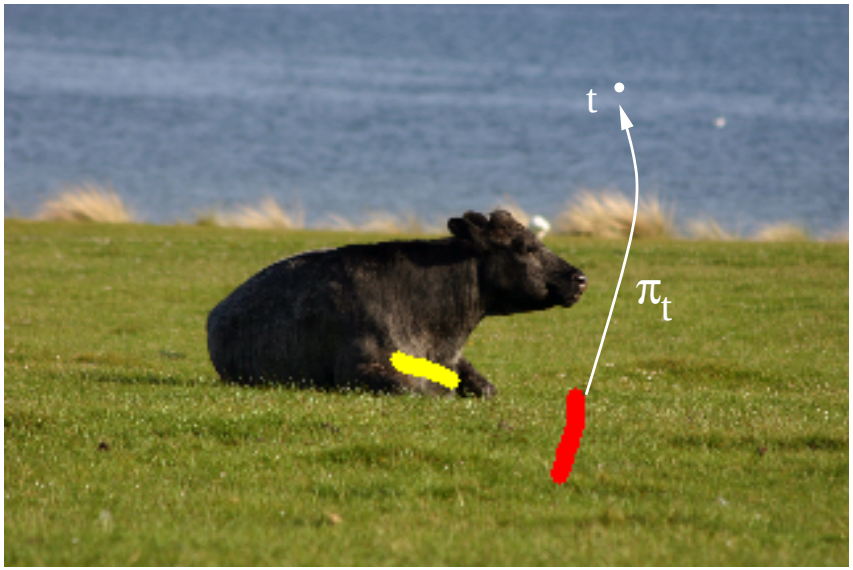


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- Image with internal and external markers.
- Arc-weight image.
- Optimum-paths to foreground pixels.
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- Segmentation result.

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show video-iftsc.gif

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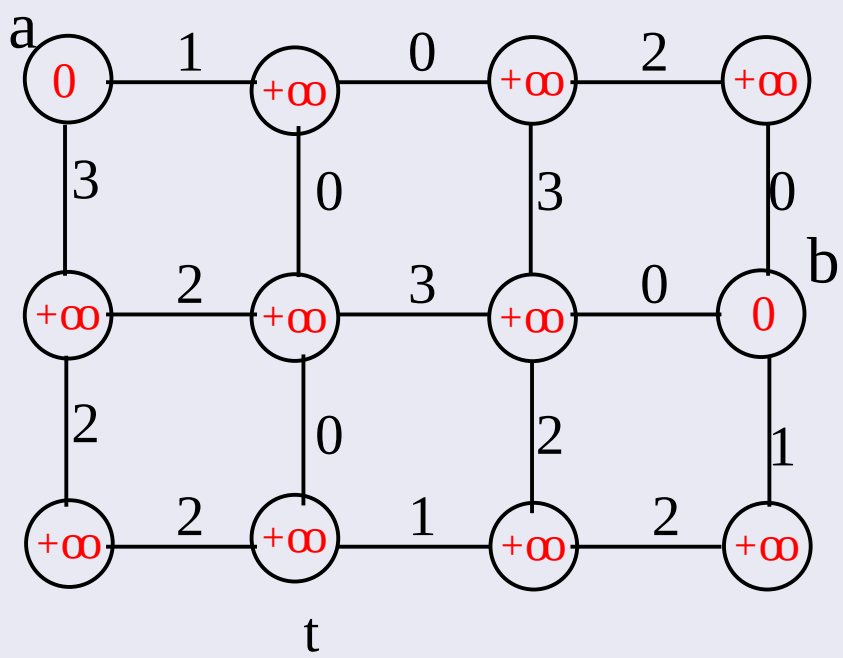
- a path $\pi_t = \pi_s \cdot \langle s, t \rangle$ is the extension of a path $\pi_s$ by an arc $(s, t)$, being $\pi_t = \langle t \rangle$ a trivial path, and
- the max-arc path function is defined as

$$
\begin{aligned}
f_{\max}(\langle t \rangle) &= \begin{cases} 0 & \text{if } t \in \mathcal{S} = \mathcal{S}_i \cup \mathcal{S}_e \\ +\infty & \text{otherwise} \end{cases} \\
f_{\max}(\pi_s \cdot \langle s, t \rangle) &= \max\{f_{\max}(\pi_s), w(s, t)\}.
\end{aligned}
$$

# Connectivity functions

Consider, for example, a 4-neighborhood graph, path function $f_{\max}$ and two seeds $\mathcal{S}_i = \{a\}$ and $\mathcal{S}_e = \{b\}$.




*4−neighborhood graph*

Paths are represented in backwards.

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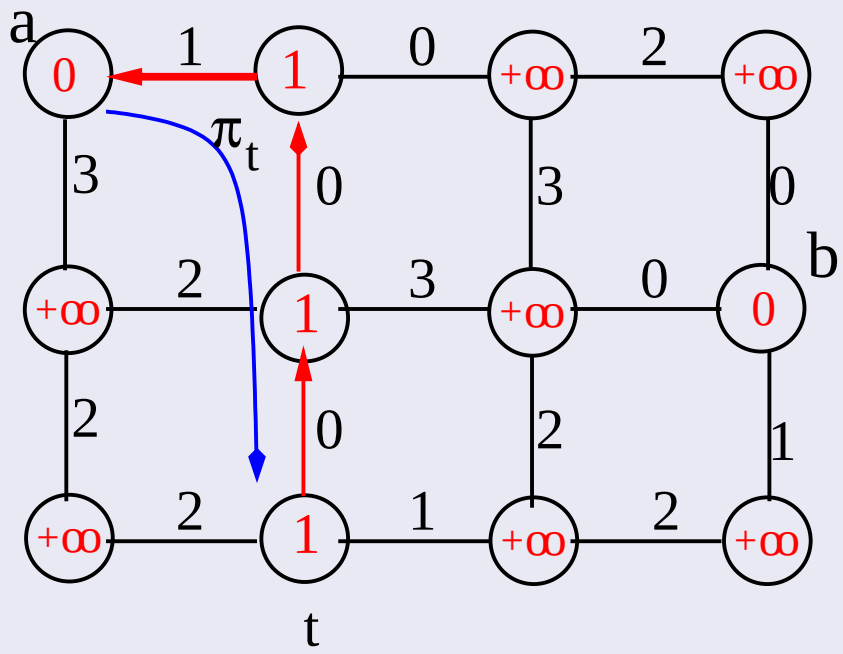


$$f_{max}(\pi_t) = 1 \quad \text{for} \quad R(\pi_t) = a$$

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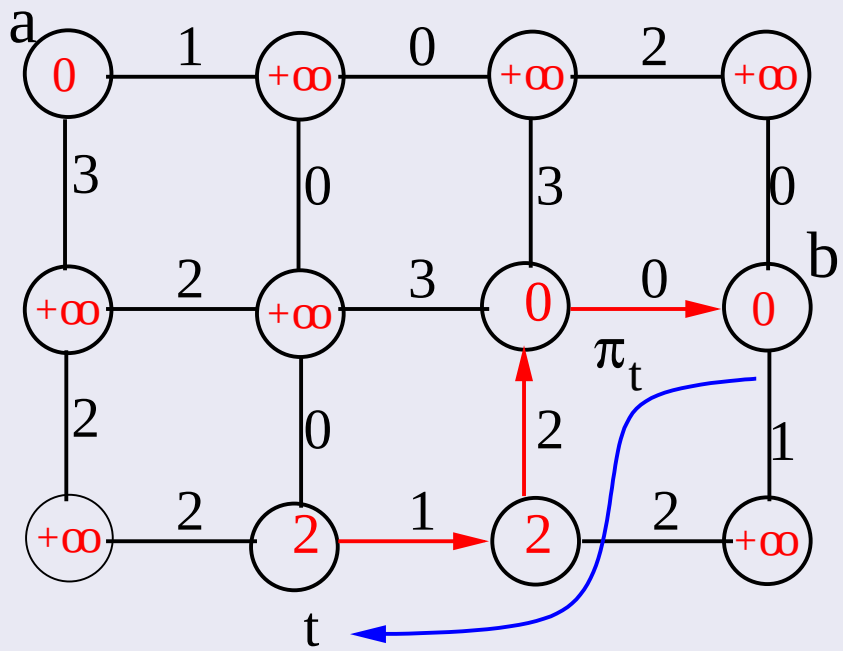



$$f_{max}(\pi_t) = 2 \quad \text{for} \quad R(\pi_t) = b$$

Paths are represented in backwards.

# Connectivity functions

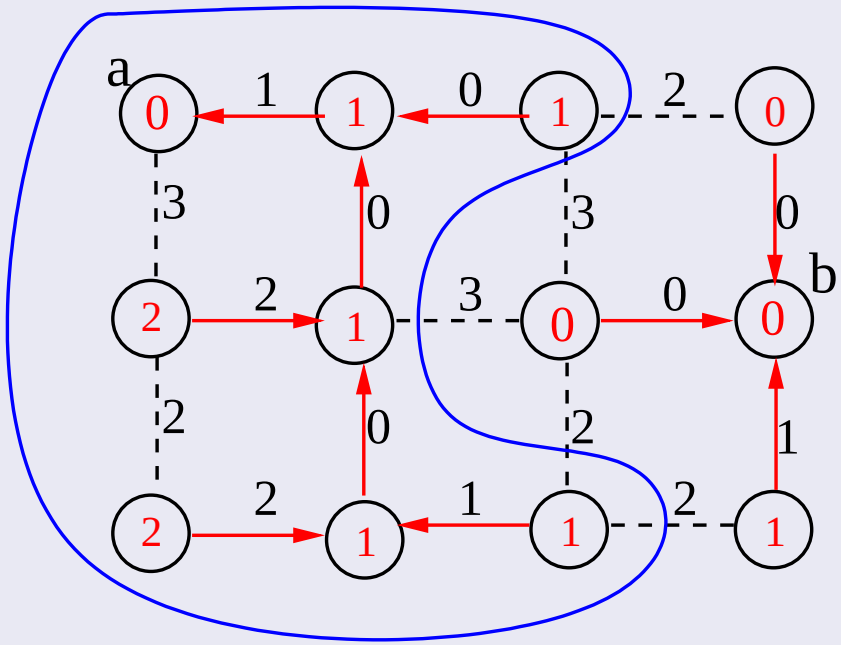
The segmentation essentially minimizes a connectivity map

$$V(t) = \min_{\forall \pi_t \in \Pi(\mathcal{N},\mathcal{A},t)} \{f_{\max}(\pi_t)\}$$

by considering the set $\Pi(\mathcal{N}, \mathcal{A}, t)$ of all paths with terminus $t$ and function $f_{\max}$.

# Connectivity function

The IFT algorithm solves this problem by computing an optimum-path forest $P$ in $(\mathcal{N}, \mathcal{A})$ — a predecessor map with no cycles, containing all optimum paths from a root set $\mathcal{R}$, which in this case is $\mathcal{S} = \mathcal{S}_i \cup \mathcal{S}_e$.



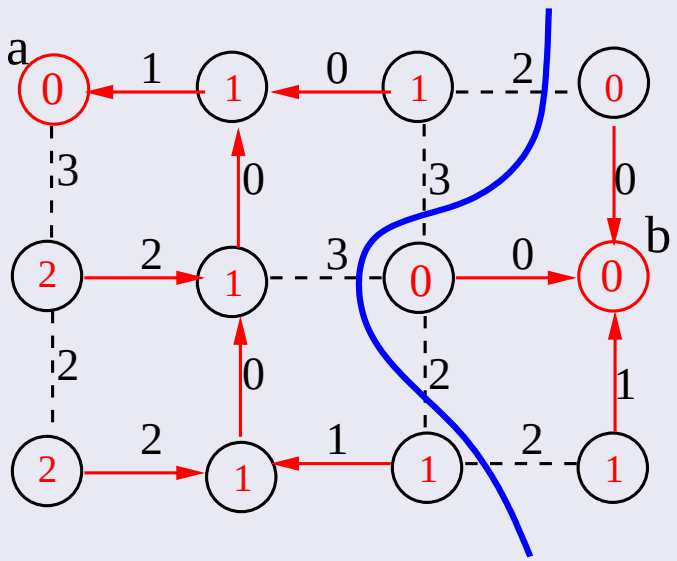

The object is defined by the optimum forest for $f_{\max}$ rooted in $\mathcal{S}_i$.

# Connectivity function

An optimum-path forest for $f_{\max}$ also provides the graph cut whose minimum arc weight

$$\min_{\forall (s,t) \in \mathcal{A}, R(\pi_s)=a, R(\pi_t)=b} w(s,t)$$

is maximum, considering all possible cuts between $a$ and $b$ [5, 7].



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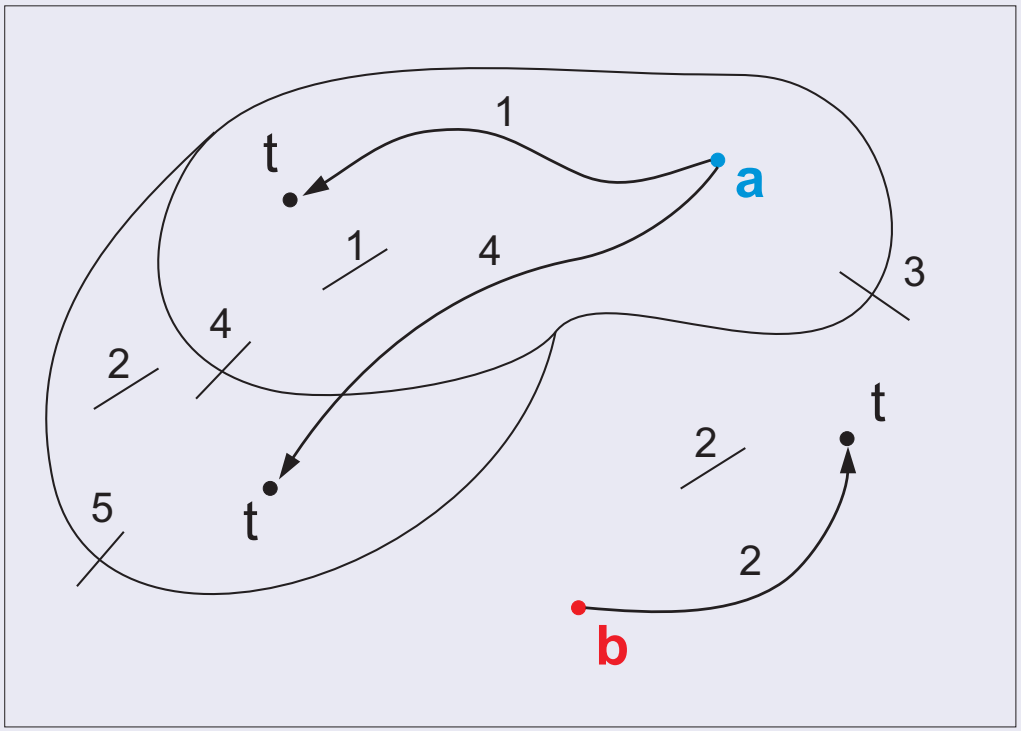
Indeed, the graph cut obtained with $f_{max}$ is piecewise optimum[5].

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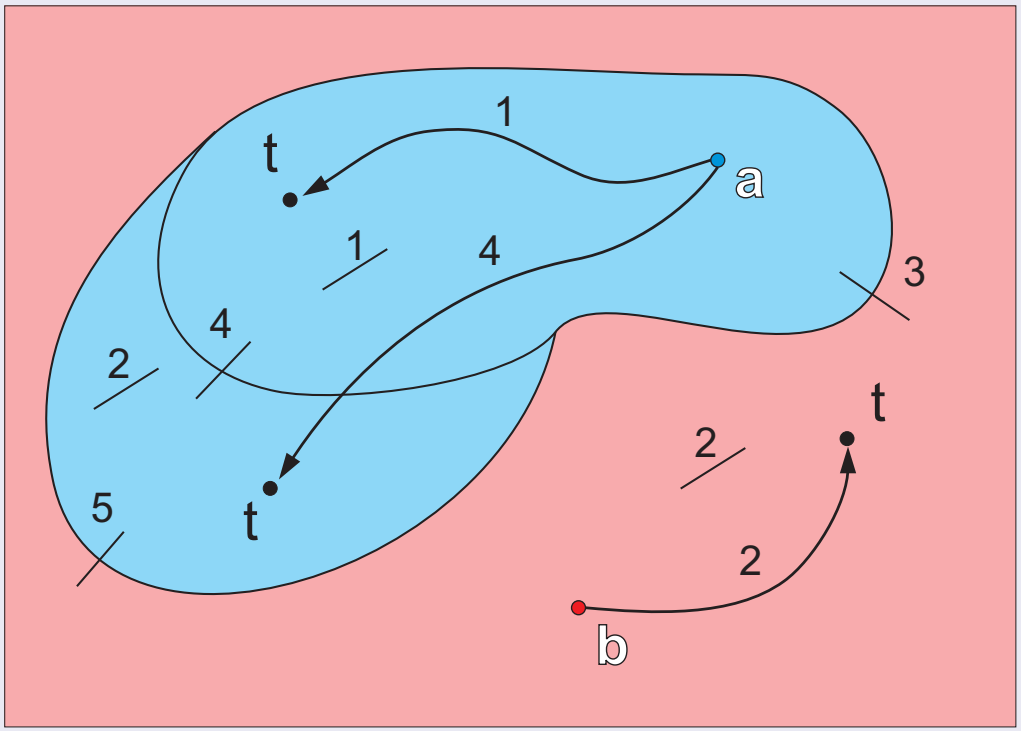


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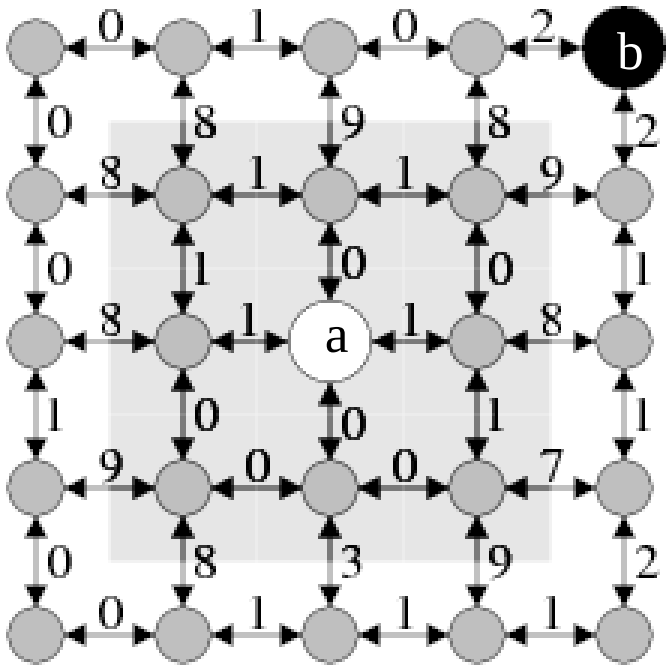
- Essentially the minima in $V_0(t)$ compete among themselves and some of them become roots in $\mathcal{R}$, being also minima in $V(t)$. show video-iftsc-computation.gif

# Path propagation

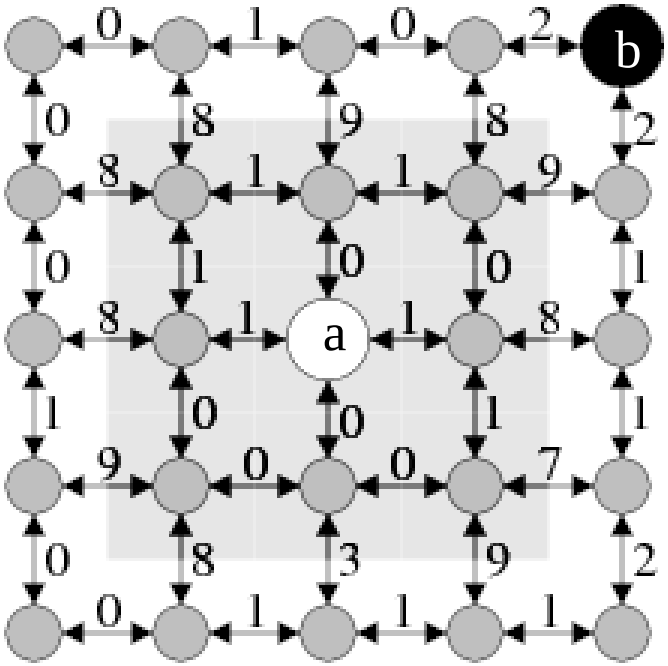
Consider the optimum path propagation for $f_{\max}$ from $\mathcal{S} = \{a, b\}$ in the 4-neighborhood graph below.

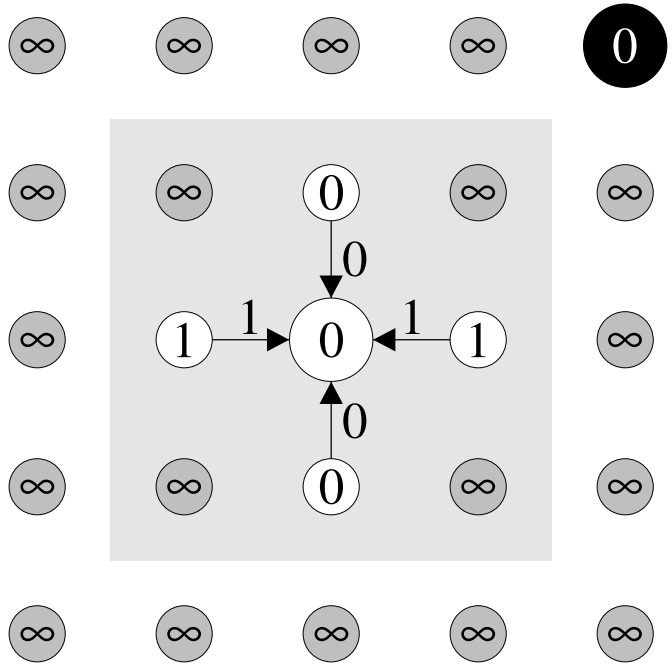



Object and background are separated by the arcs with higher weights.

# Path propagation

From iteration 1 to 5, iteration 12, 20, and 25.



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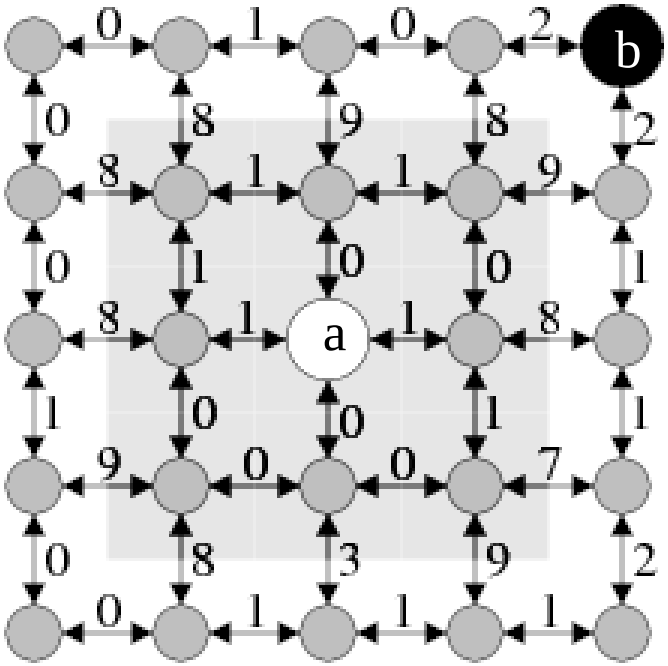


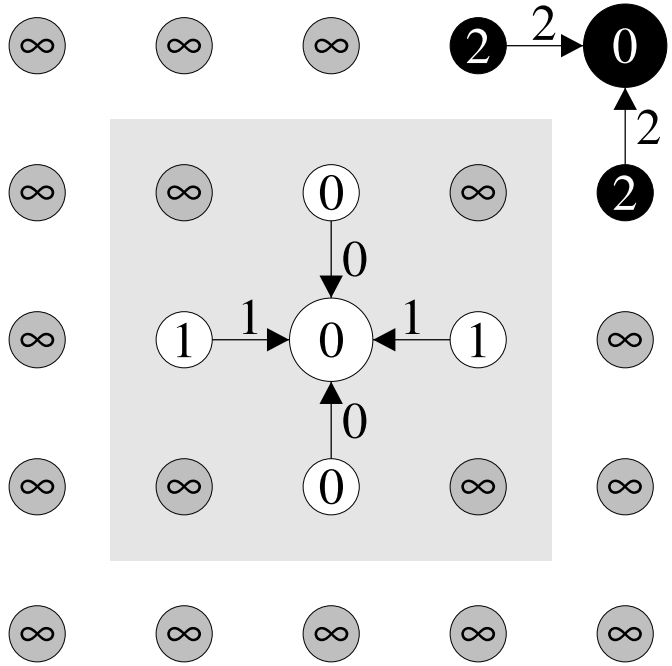

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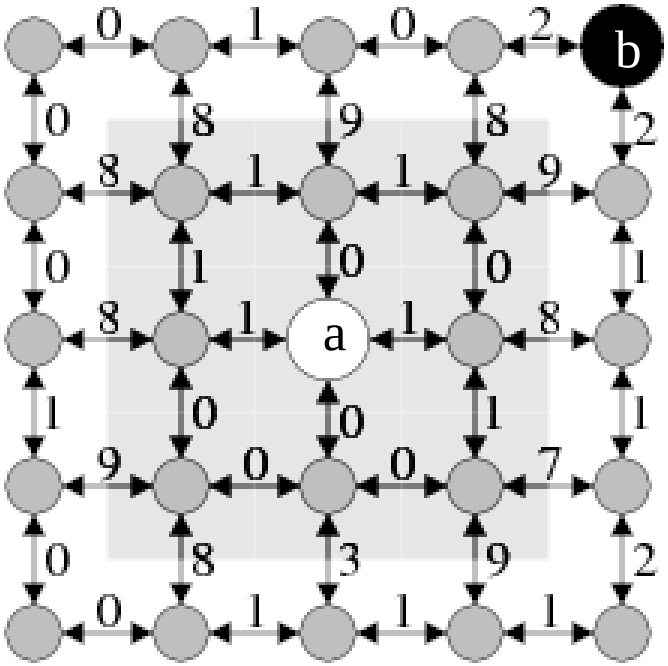


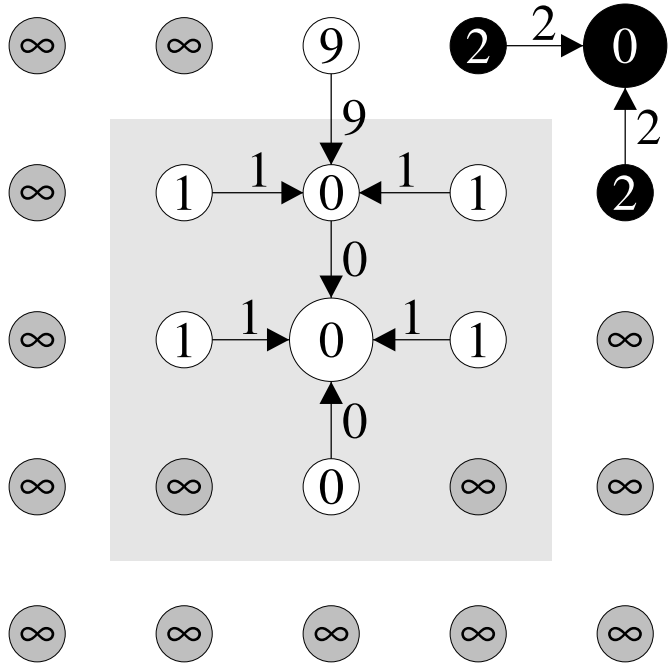

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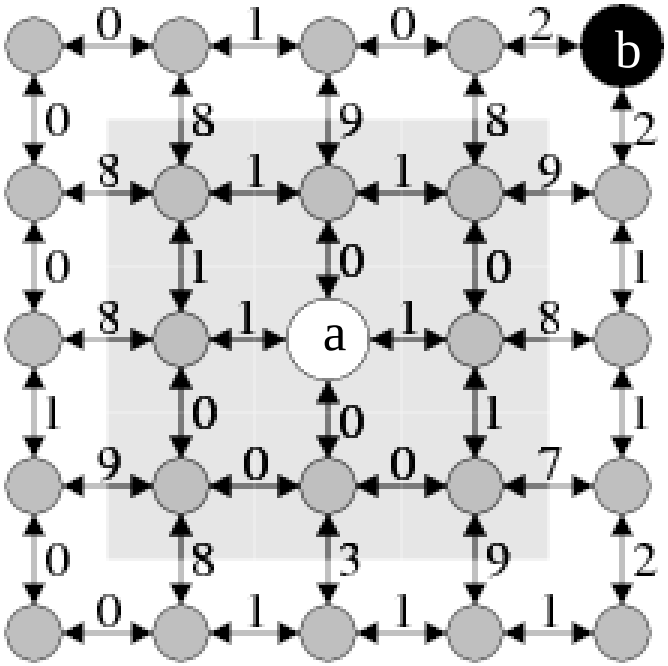


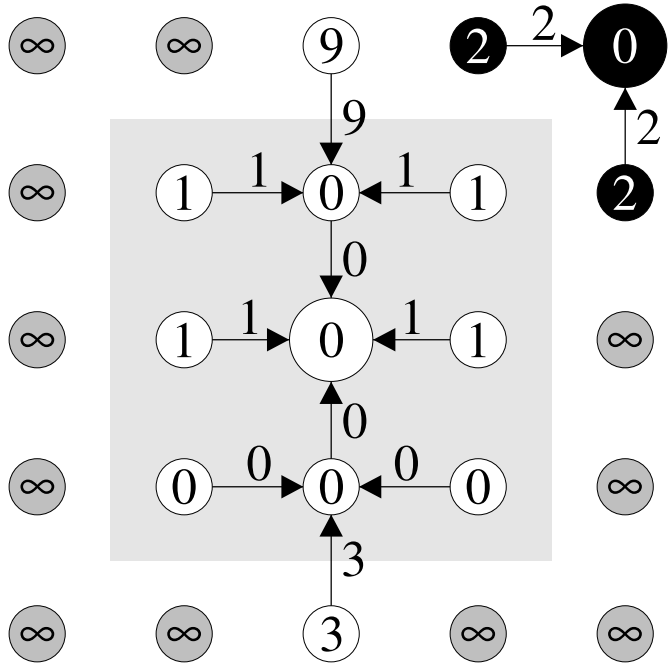

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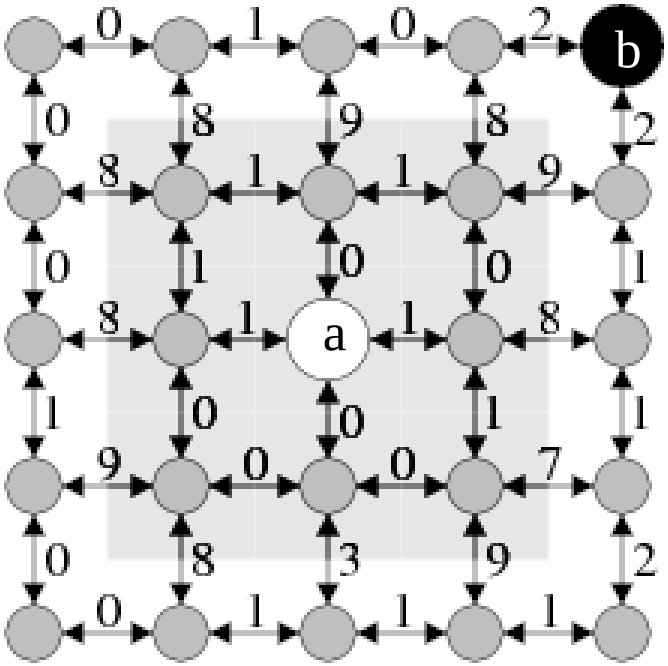


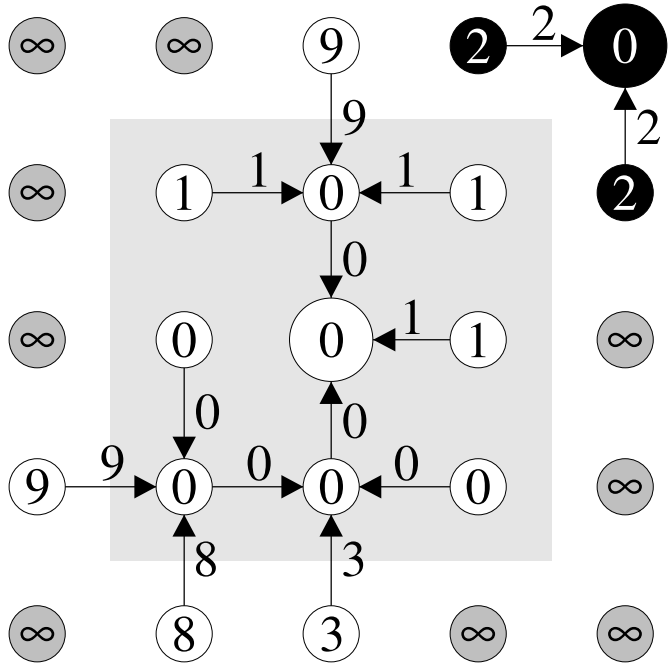

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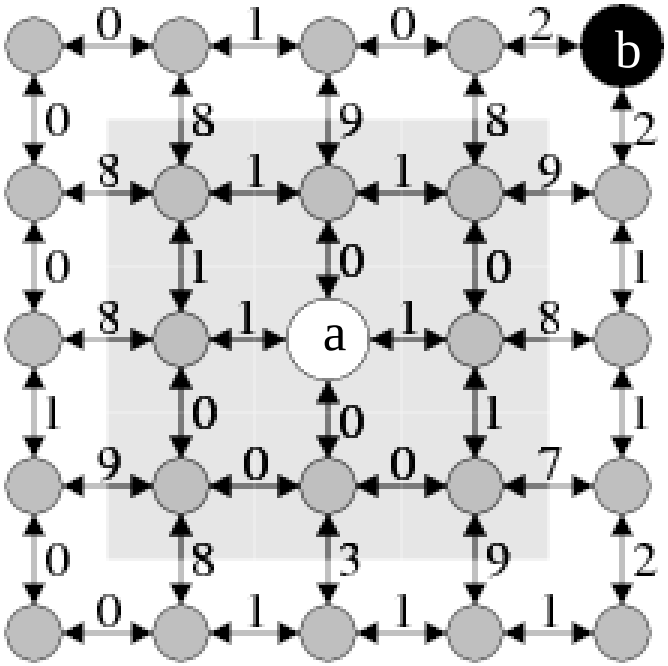


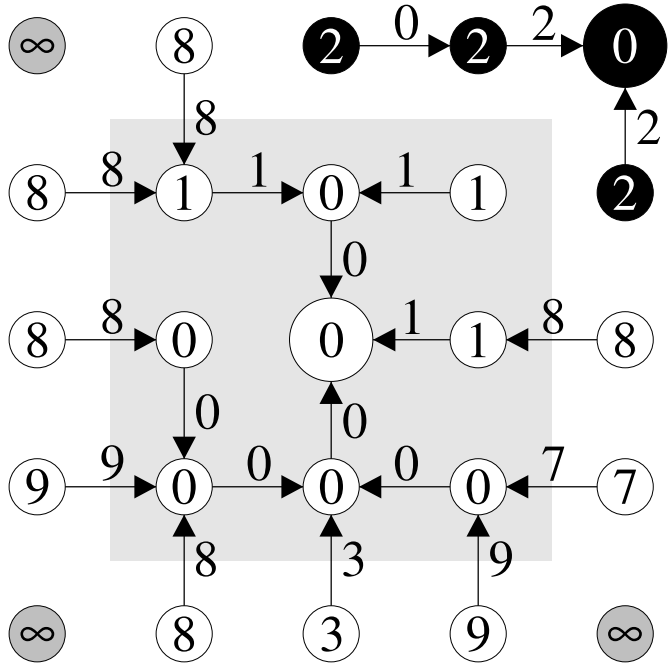

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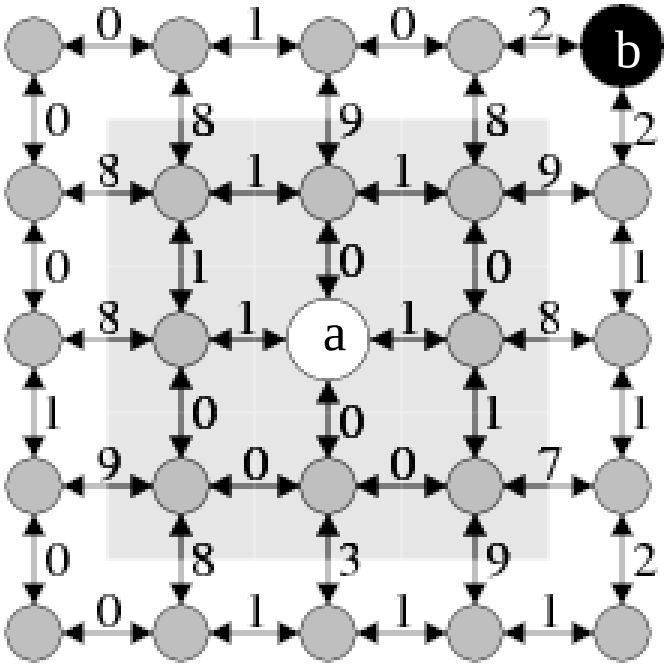


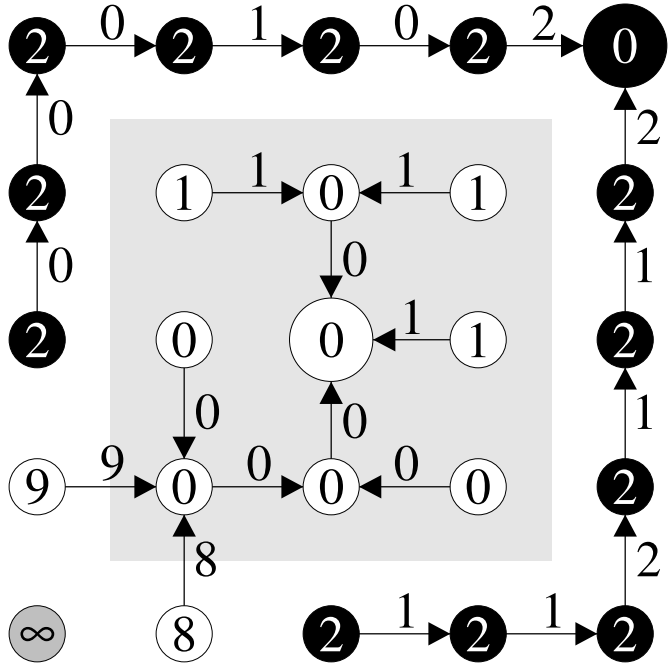

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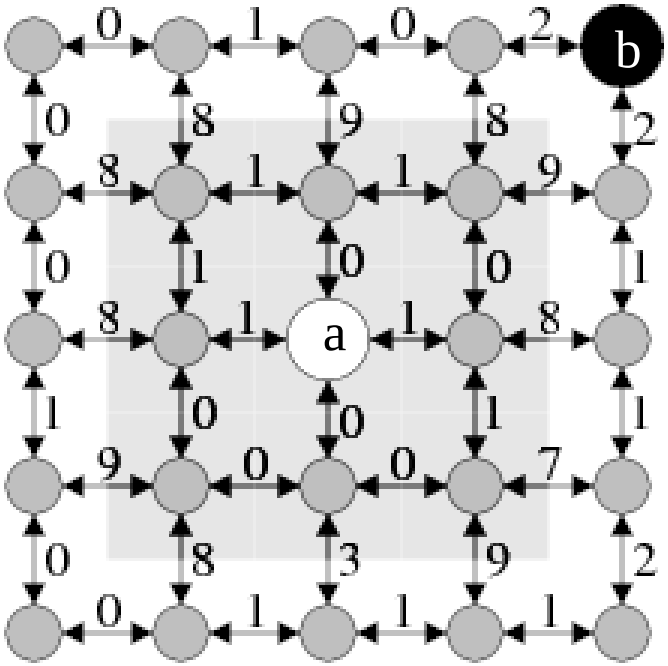


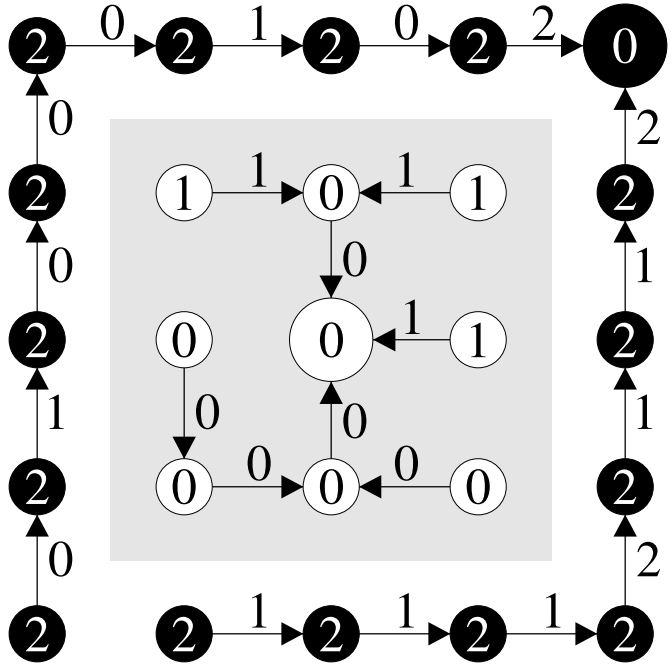

# Information propagation

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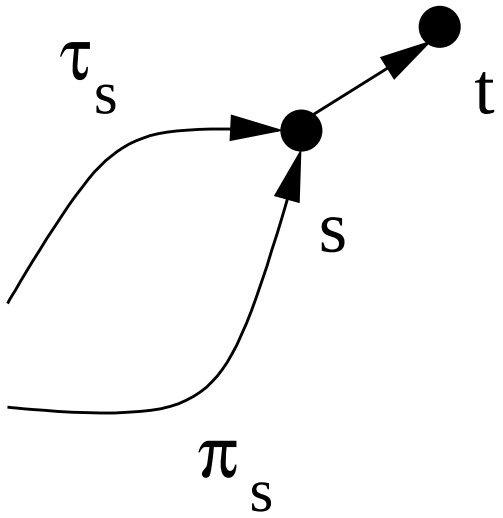
- Its correctness requires smooth connectivity functions.

# Information propagation

A path function is smooth when for every node $t$, there exists at least one optimum path $\pi_t$ which is either trivial or has the form $\pi_s \cdot \langle s, t \rangle$ where:

1. $f(\pi_s) \leq f(\pi_t)$.
2. $\pi_s$ is optimum.
3. For any optimum path $\tau_s$, $f(\tau_s \cdot \langle s, t \rangle) = f(\pi_t)$.

These conditions are applied to only optimum paths.



# The general IFT algorithm

## Algorithm

– General IFT algorithm

1. For each $t \in \mathcal{N}$, do
2. $\quad$ Set $P(t) \leftarrow nil$, $L(t) \leftarrow t$ and $V(t) \leftarrow f(\langle t \rangle)$.
3. $\quad$ If $V(t) \neq +\infty$, then insert $t$ in $Q$.
4. While $Q$ is not empty, do
5. $\quad$ Remove from $Q$ a spel $s$ such that $V(s)$ is minimum.
6. $\quad$ For each $t \in \mathcal{A}(s)$ such that $V(t) > V(s)$, do
7. $\quad\quad$ Compute $tmp \leftarrow f(\pi_s \cdot \langle s, t \rangle)$.
8. $\quad\quad$ If $tmp < V(t)$, then
9. $\quad\quad\quad$ If $V(t) \neq +\infty$, remove $t$ from $Q$.
10. $\quad\quad\quad$ Set $P(t) \leftarrow s$, $V(t) \leftarrow tmp$, $L(t) \leftarrow L(s)$.
11. $\quad\quad\quad$ Insert $t$ in $Q$.

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- Line 7 can be simplified to $tmp \leftarrow \max\{V(s), w(s,t)\}$ in the case of $f_{\max}$.
- The dual operation $V(t) = \max_{\forall \pi_t \in \Pi(\mathcal{N},\mathcal{A},t)}\{f_{\min}(\pi_t)\}$ requires: $V(t) \neq -\infty$ in Lines 3 and 9, $V(s)$ is maximum in Line 5, $V(t) < V(s)$ in Line 6, and $tmp > V(t)$ in Line 8.

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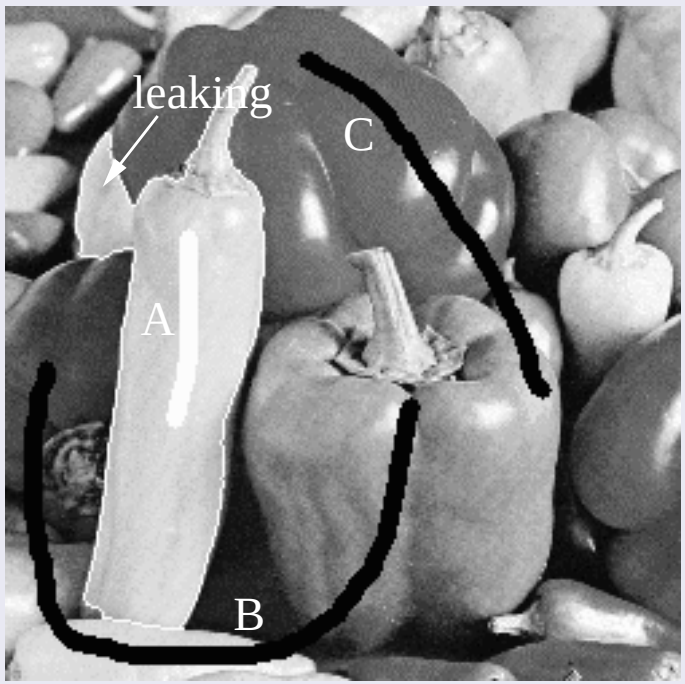
- Early termination is possible whenever $s$ is a destination node [2] or when $V(s)$ is greater than a given threshold [24].
- Later, whenever necessary, the remaining optimum paths $\pi_t$ with $V(t) \geq V(s)$ can be obtained incrementally from the nodes in $Q$.

# Important remarks and variants

For a fixed path function $f$, the optimum forest can be updated in a differential way whenever we add/remove root nodes [3].

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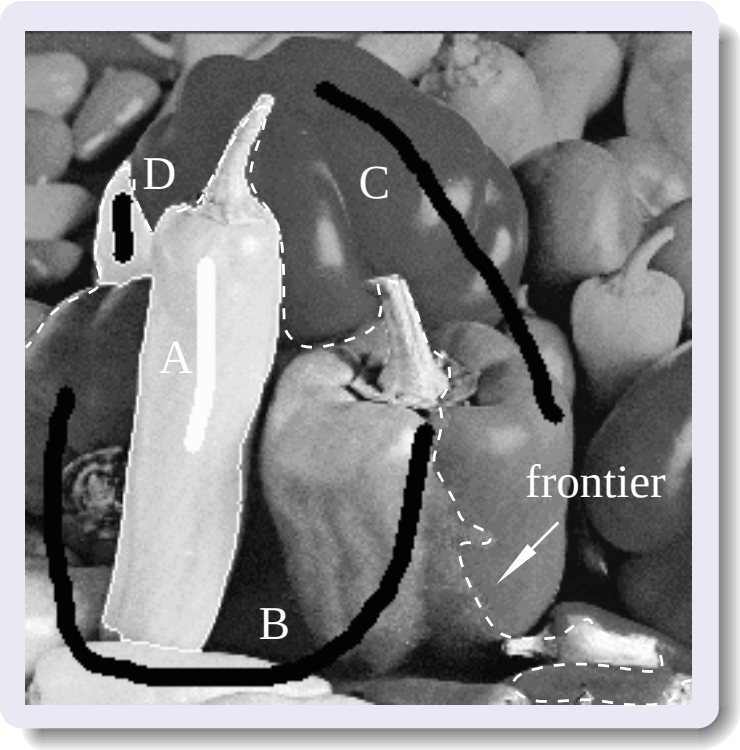


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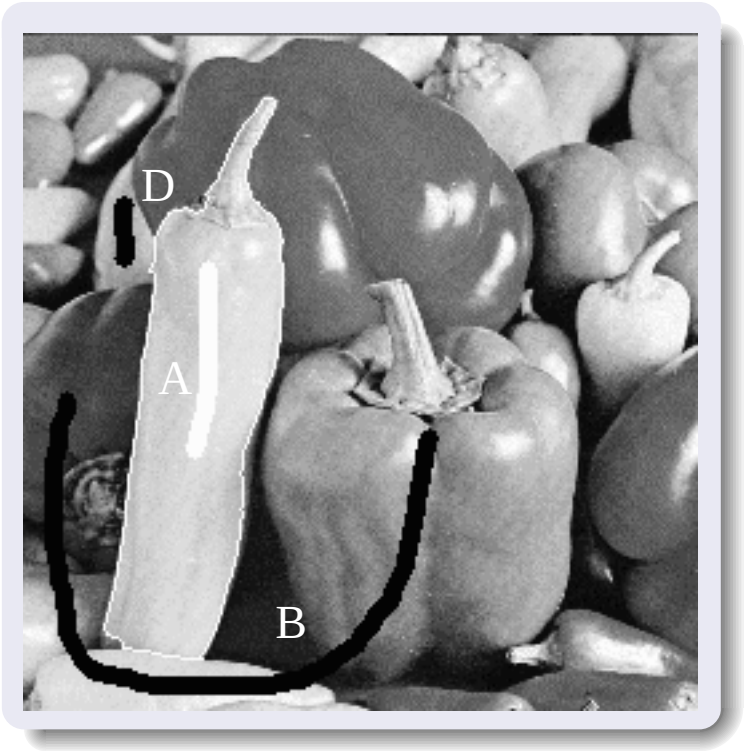


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- Result of segmentation.

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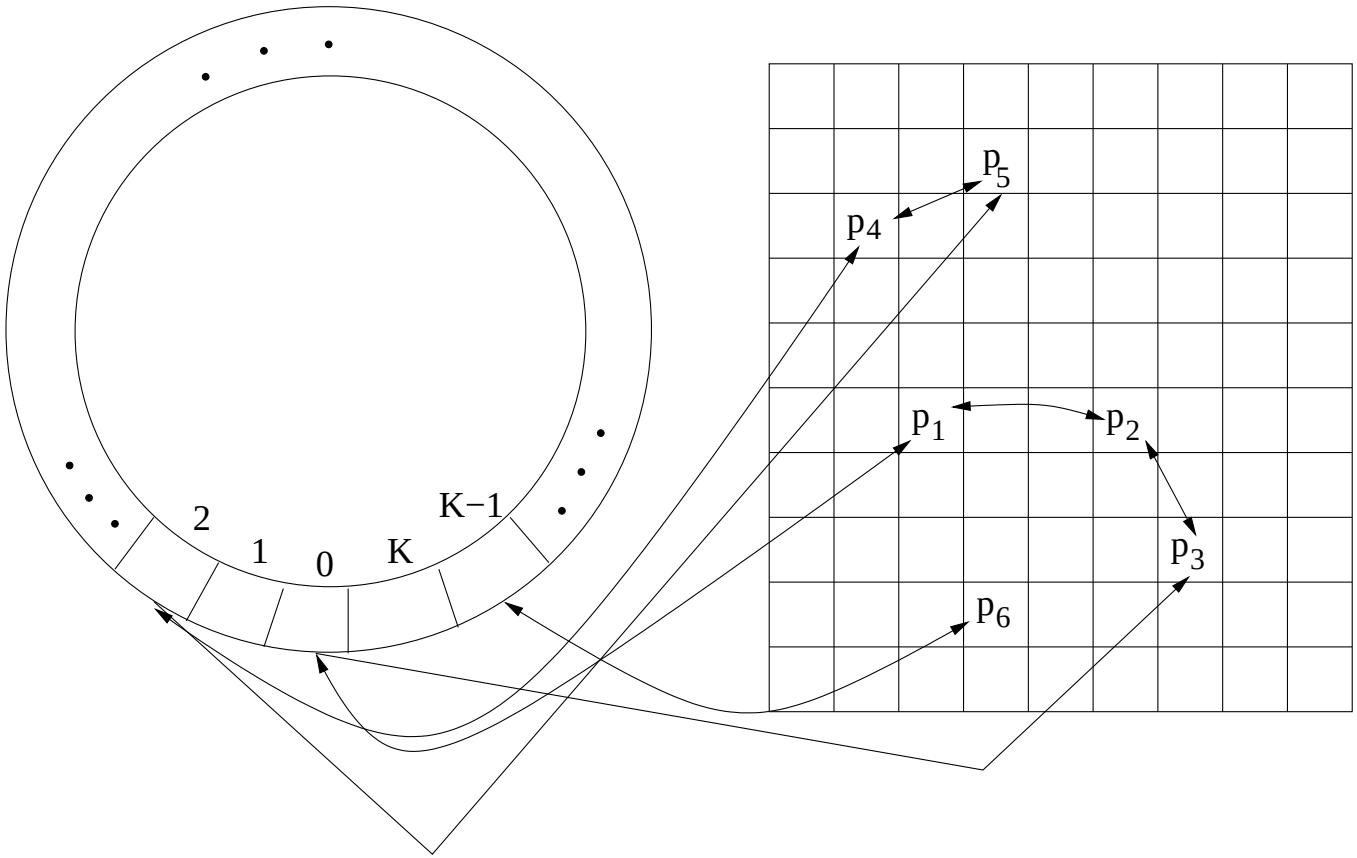
- Its running time is $O(|\mathcal{N}|)$, when $f(\pi_s \cdot \langle s, t \rangle) - f(\pi_s) \in [0..K]$, $K \ll |\mathcal{N}|$, are integers, the graph is sparse, and $Q$ uses bucket sort [2].

# Priority queue




Nodes $t$ are inserted in bucket $V(t)\%(K+1)$ (left), forming $K+1$ lists (right). The property $f(\pi_s \cdot \langle s,t \rangle) - f(\pi_s) \in [0..K]$ guarantees that nodes with different values are never in a same bucket.

# Conclusion

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- It can be efficiently implemented in time proportional to the number of nodes for most image transformations.
- A demonstration C code is available at `http://code.google.com/p/ift-demo/`.

# Next lecture

- The Image Foresting Transform.
- Interactive and automatic segmentation methods.
- Clustering and classification.
- Connected filters.

Thanks for your attention

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